

Linear Programming

Optimization problem:

$$\max_{x \in \mathbb{R}, y \in \mathbb{R}}$$

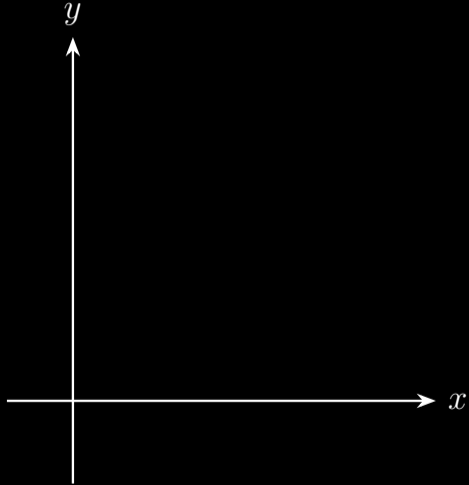
$$x + y$$

$$\text{s.t. } x, y \geq 0$$

$$3x + 2y \leq 18$$

$$-x + 2y \geq 2$$

$$x \leq 3$$



Linear Programming

Optimization problem:

$$\max_{x \in \mathbb{R}, y \in \mathbb{R}}$$

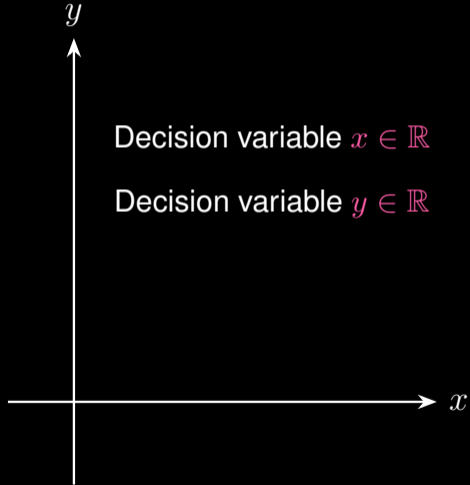
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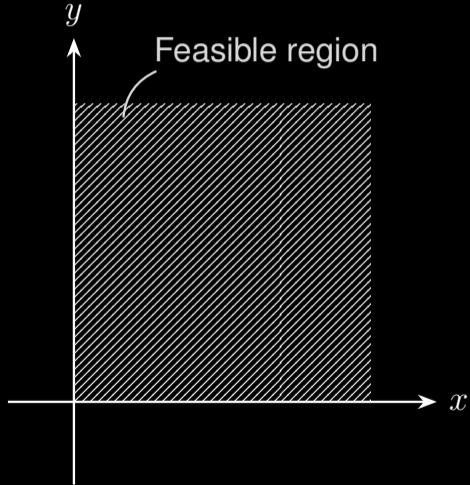
$$x \leq 3$$



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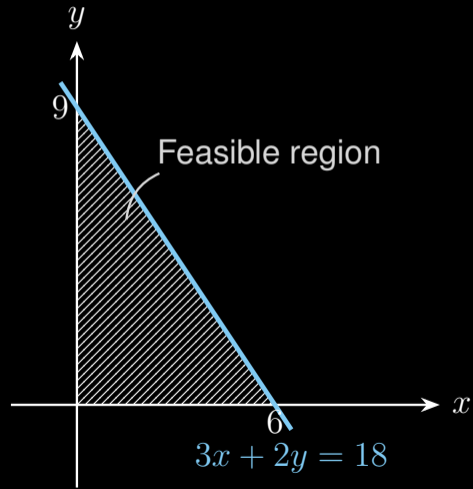
$$\begin{aligned} \max_{x \in \mathbb{R}, y \in \mathbb{R}} \quad & x + y \\ \text{s.t.} \quad & x, y \geq 0 \end{aligned}$$



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Linear Programming

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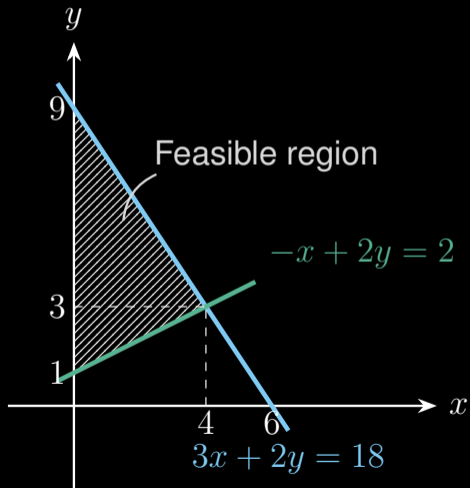
$$\max_{x \in \mathbb{R}, y \in \mathbb{R}}$$

$$x + y$$

$$\text{s.t. } x, y \geq 0$$

$$3x + 2y \leq 18$$

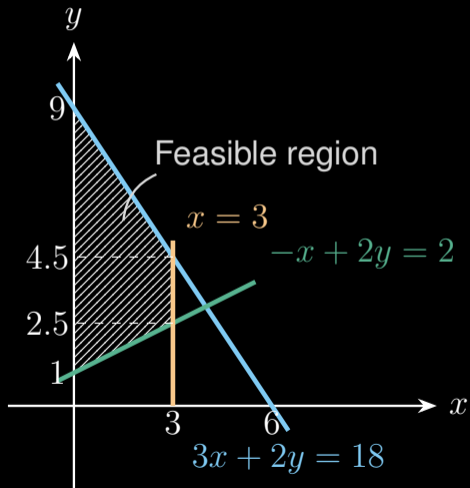
$$-x + 2y \geq 2$$



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Linear Programming

Optimization problem:

$$\max_{x \in \mathbb{R}, y \in \mathbb{R}} x + y$$

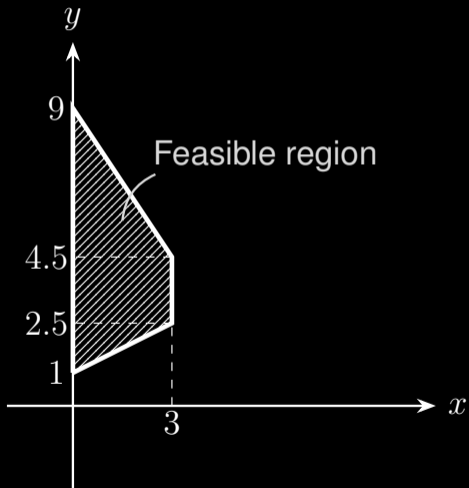
$$\text{s.t. } x, y \geq 0$$

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$$-x + 2y \geq 2$$

$$x \leq 3$$

The shaded area contains all (x, y) pairs that satisfy every constraint simultaneously.

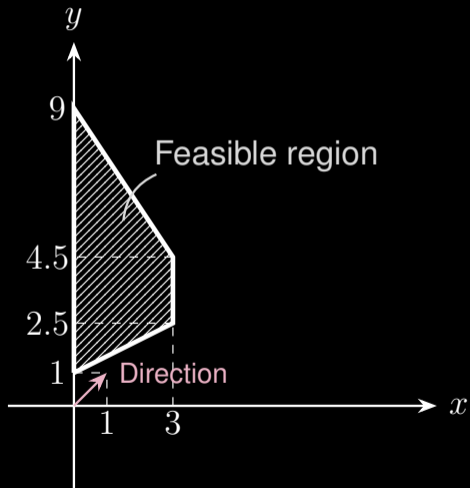


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Direction vector $(1, 1)$

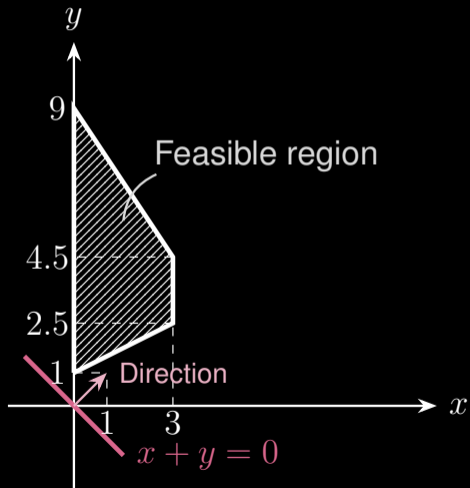


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Direction vector $(1, 1)$

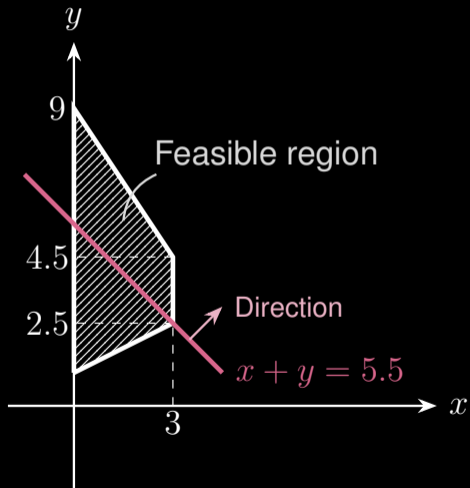


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$$x + y$$

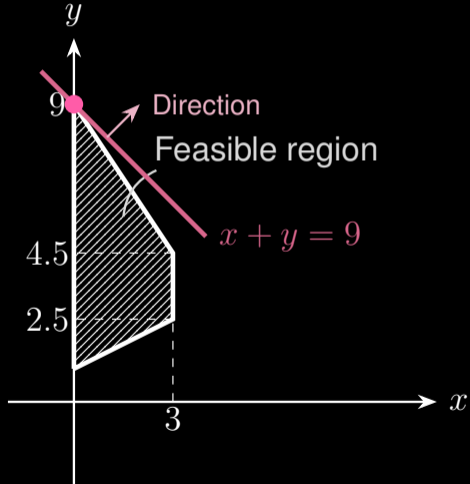
$$\text{s.t. } x, y \geq 0$$

$$3x + 2y \leq 18$$

$$-x + 2y \geq 2$$

$$x \leq 3$$

Optimal solution: $x = 0, y = 9$

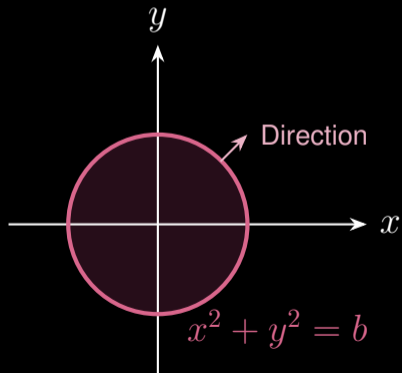
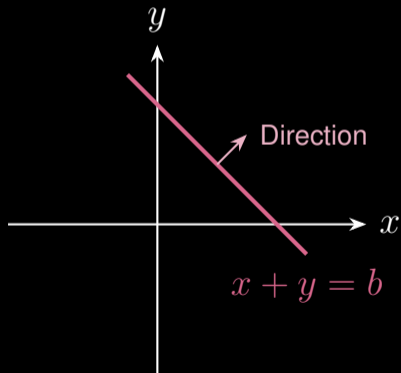


How about quadratic objective function?

$$\max x + y$$

→

$$\max x^2 + y^2$$



Quadratic Programming

Optimization problem:

$$\max_{x \in \mathbb{R}, y \in \mathbb{R}}$$

$$x^2 + y^2$$

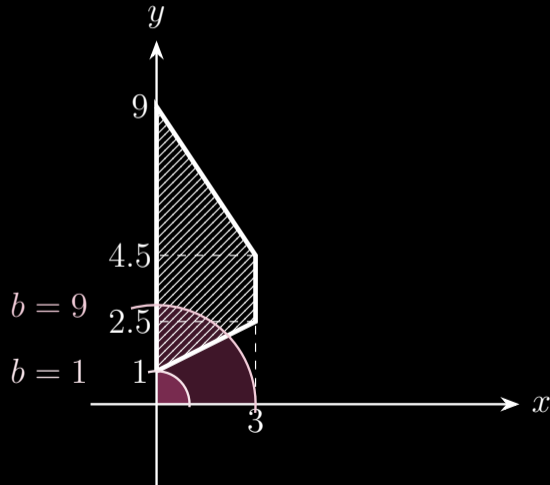
$$\text{s.t. } x, y \geq 0$$

$$3x + 2y \leq 18$$

$$-x + 2y \geq 2$$

$$x \leq 3$$

Objective function $x^2 + y^2 = b$



Quadratic Programming

Optimization problem:

$$\max_{x \in \mathbb{R}, y \in \mathbb{R}}$$

$$x^2 + y^2$$

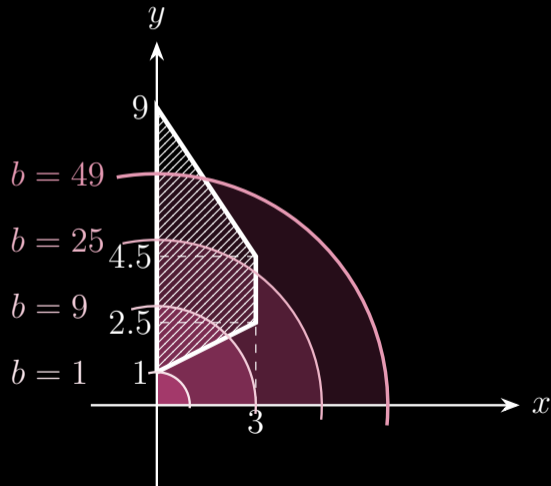
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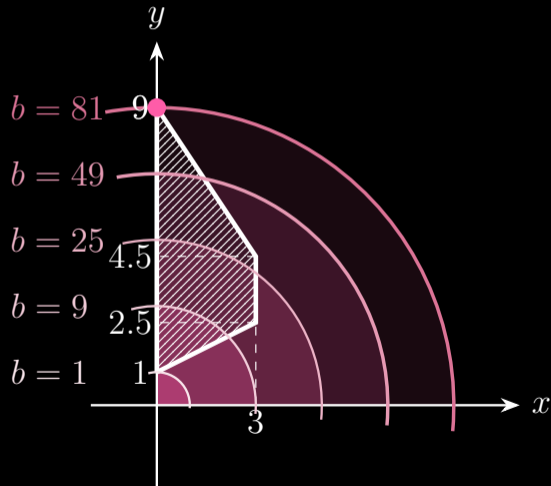


Quadratic Programming

Optimization problem:

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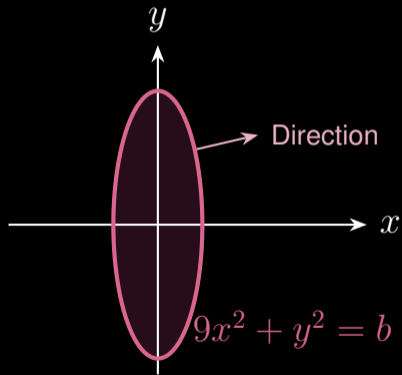
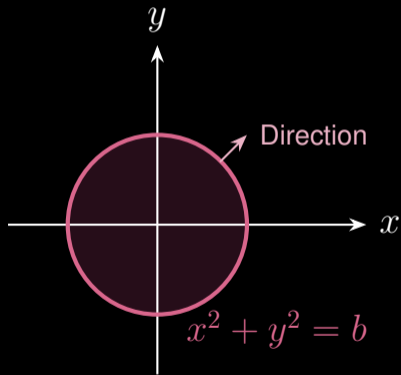
Optimal solution: $x = 0, y = 9$



$$\max x^2 + y^2$$

→

$$\max 9x^2 + y^2$$

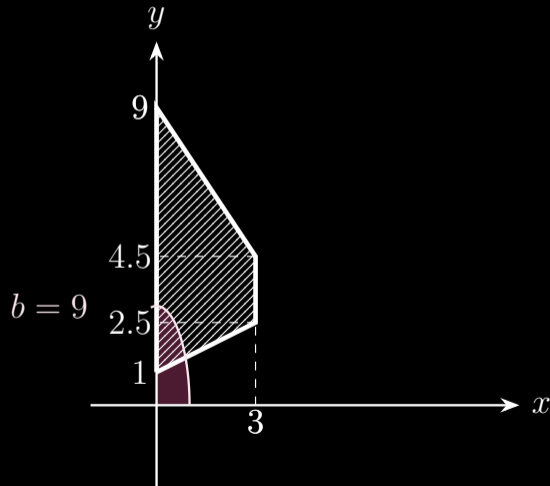


Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x \in \mathbb{R}, y \in \mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective function $9x^2 + y^2 = b$

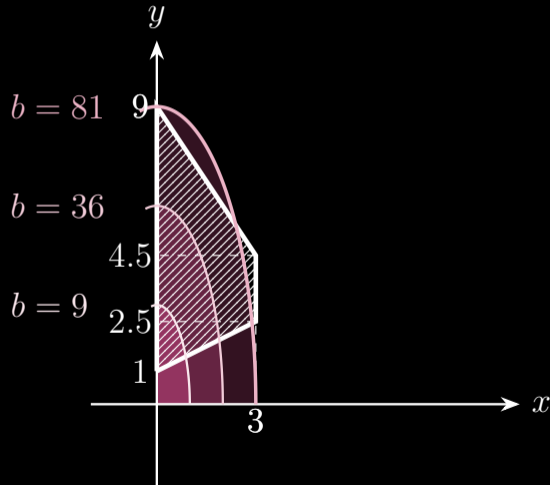


Quadratic Programming

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Objective function $9x^2 + y^2 = b$

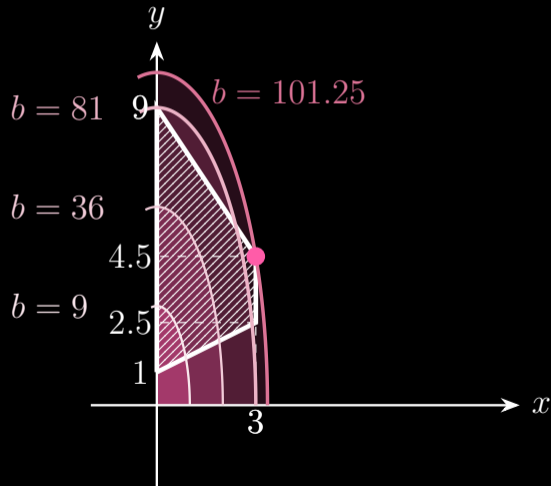


Quadratic Programming

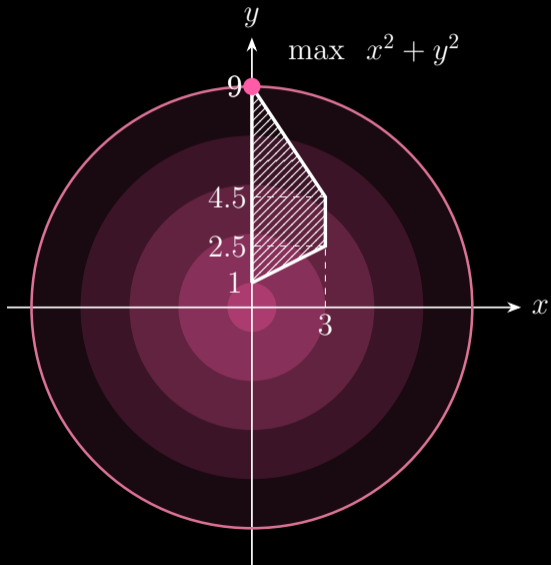
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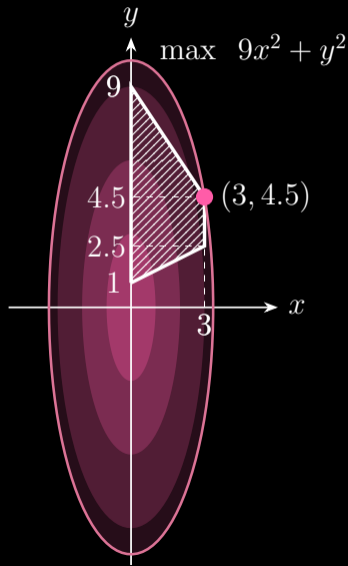
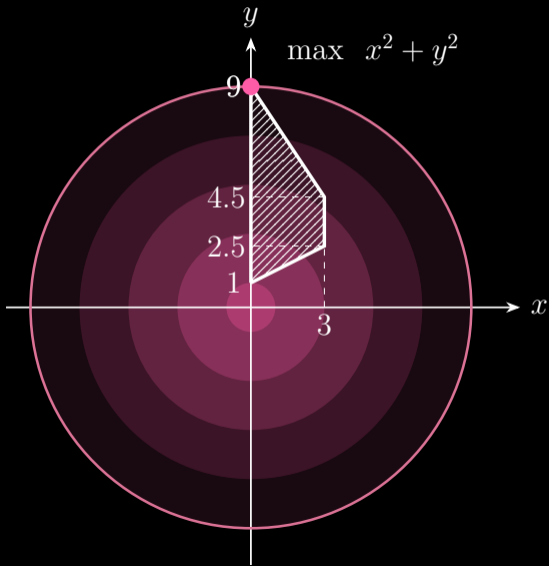
Optimal solution: $x = 3, y = 4.5$



Different objectives \rightarrow different optimal solutions



Different objectives \rightarrow different optimal solutions



Thanks for your attention!

About me:

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 How to reach me: chenxy346@gmail.com