What Is the Orthogonal Procrustes Problem?

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\begin{aligned} \min_{\boldsymbol{F}} \quad & \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2 \\ \text{s.t.} \quad & \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}}_{\text{orthogonal}} \end{aligned}$$

is

$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

Python implementation:

```python
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

What Is the Orthogonal Procrustes Problem?

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\min_{\boldsymbol{F}} \quad \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2 \qquad \text{s.t.} \quad \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}}_{\text{orthogonal}}$$

is

$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

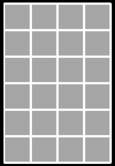
Python implementation:

```
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

① Singular value decomposition on $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$

$\boldsymbol{Q} \in \mathbb{R}^{m \times r}$



=

What Is the Orthogonal Procrustes Problem?

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\begin{aligned} \min_{\boldsymbol{F}} \quad & \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2 \\ \text{s.t.} \quad & \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}}_{\text{orthogonal}} \end{aligned}$$

is

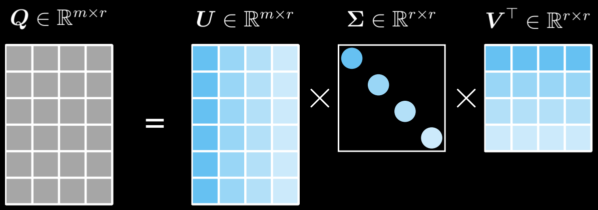
$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

Python implementation:

```
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

① Singular value decomposition on $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$



What Is the Orthogonal Procrustes Problem?

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\begin{aligned} \min_{\boldsymbol{F}} \quad & \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2 \\ \text{s.t.} \quad & \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}}_{\text{orthogonal}} \end{aligned}$$

is

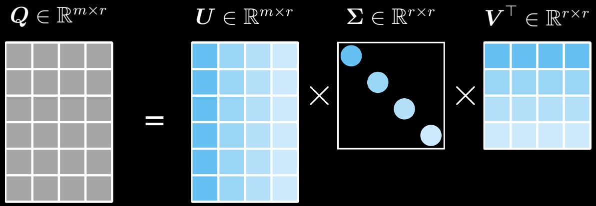
$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

Python implementation:

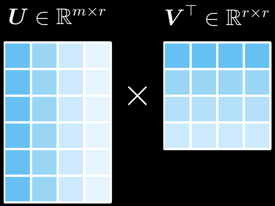
```python
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

① Singular value decomposition on $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$



② Matrix multiplication between $\boldsymbol{U}$ and $\boldsymbol{V}$



What Is the Orthogonal Procrustes Problem?

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\begin{aligned} \min_{\boldsymbol{F}} \quad & \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2 \\ \text{s. t.} \quad & \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}}_{\text{orthogonal}} \end{aligned}$$

is

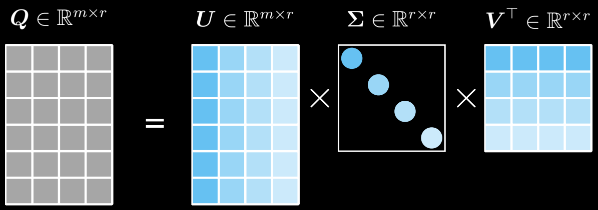
$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

Python implementation:

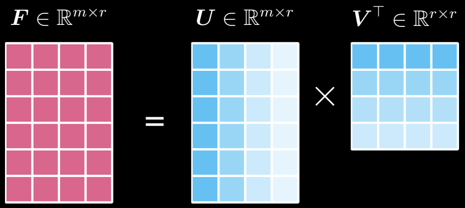
```python
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

① Singular value decomposition on $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$



② Matrix multiplication between $\boldsymbol{U}$ and $\boldsymbol{V}$



Thanks for your attention!

About me:

- Homepage: https://xinychen.github.io
- How to reach me: chenxy346@gmail.com