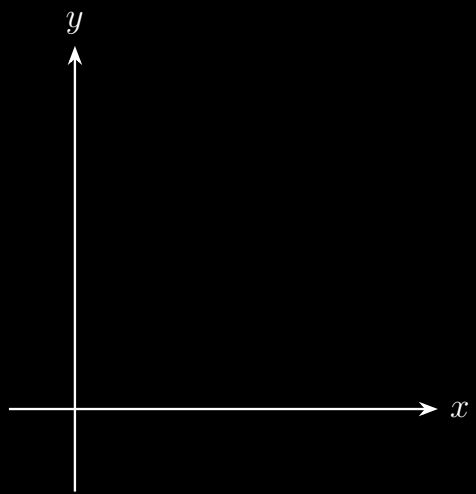
Quadratic Programming

Optimization problem:

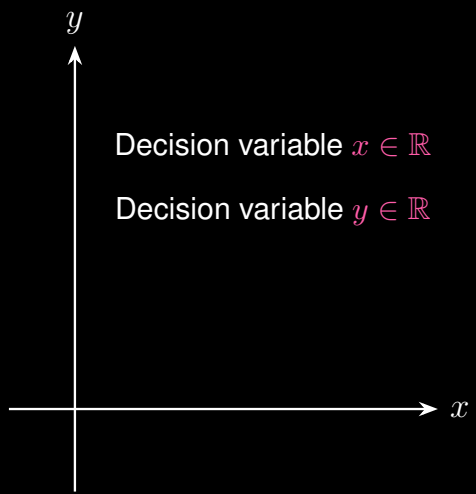
$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$



Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$



Decision variable $x \in \mathbb{R}$

Decision variable $y \in \mathbb{R}$

Quadratic Programming

Optimization problem:

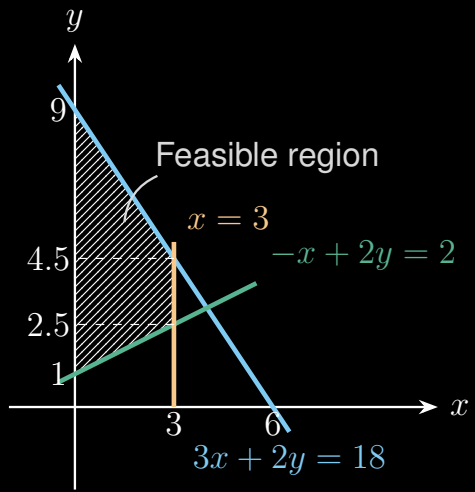
$$\max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad 9x^2 + y^2$$

$$\text{s.t.} \quad x, y \geq 0$$

$$3x + 2y \leq 18$$

$$-x + 2y \geq 2$$

$$x \leq 3$$

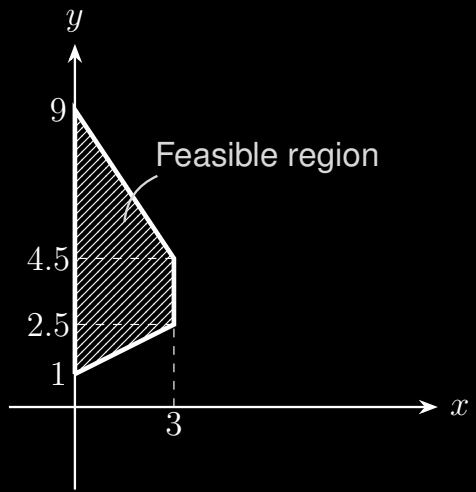


Quadratic Programming

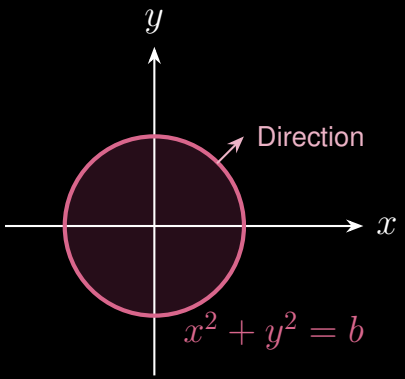
Optimization problem:

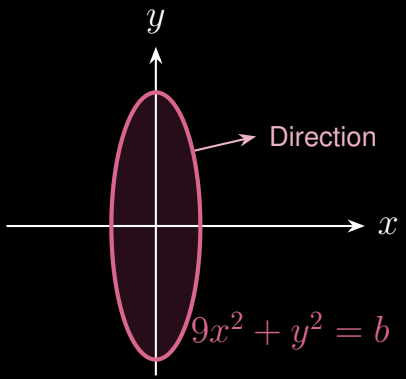
$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

The shaded area contains all (x, y) pairs that satisfy every constraint simultaneously.



$$\max_{x,\,y} \quad x^2 + y^2 \qquad \rightarrow \qquad \max_{x,\,y} \quad 9x^2 + y^2$$



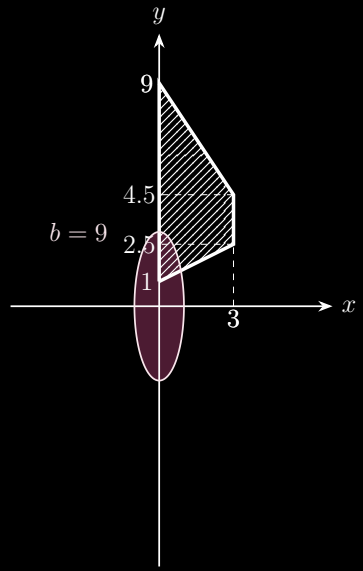


Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective $9x^2 + y^2 = b$

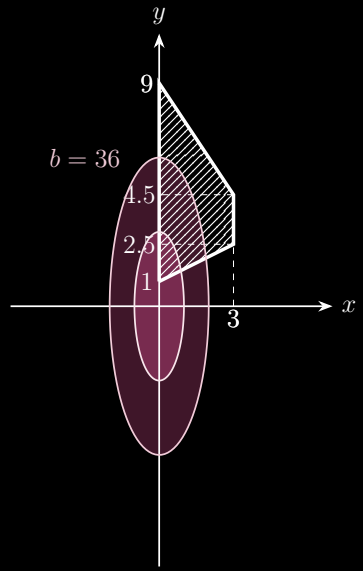


Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective $9x^2 + y^2 = b$



Quadratic Programming

Optimization problem:

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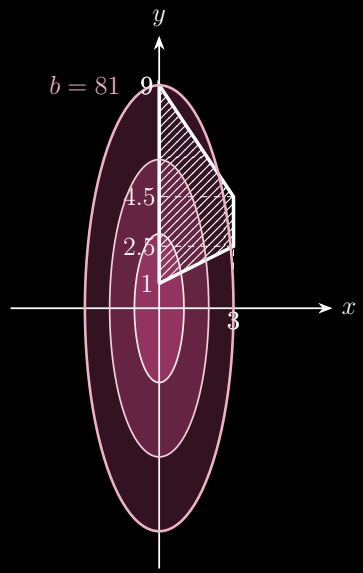
Objective $9x^2 + y^2 = b$

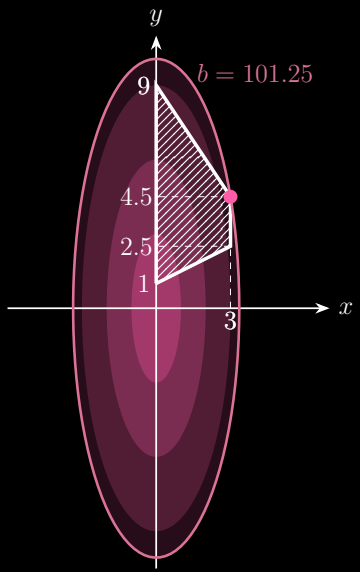


Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

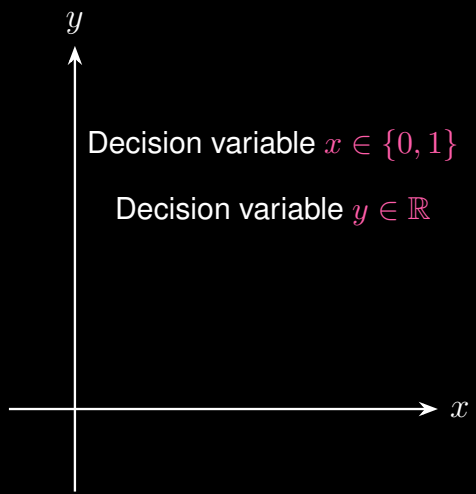
Optimal solution: $x = 3, y = 4.5$



Mixed-Integer Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

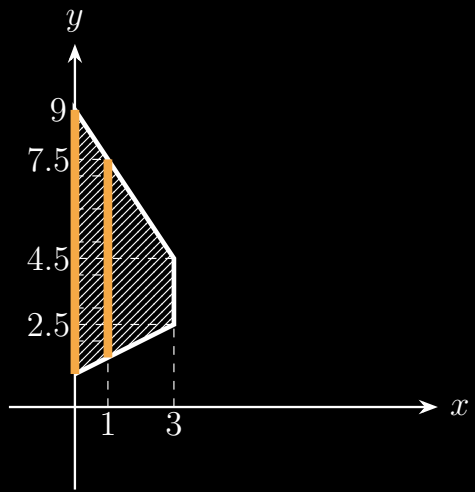


Mixed-Integer Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions ( )

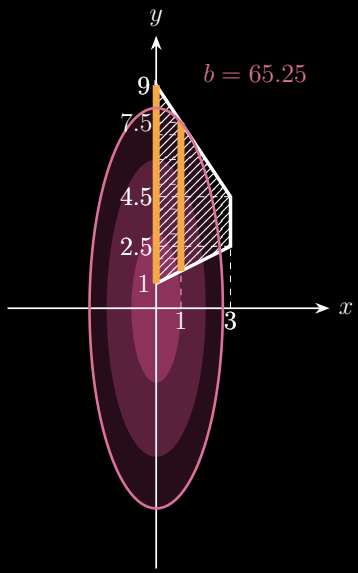


Mixed-Integer Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective $9x^2 + y^2 = b$

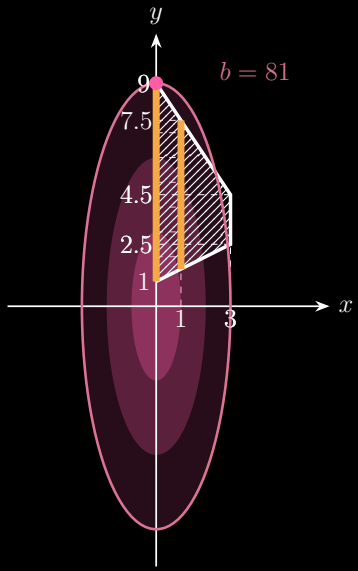


Mixed-Integer Quadratic Programming

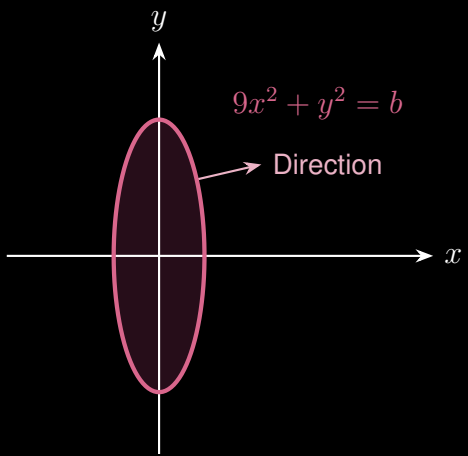
Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

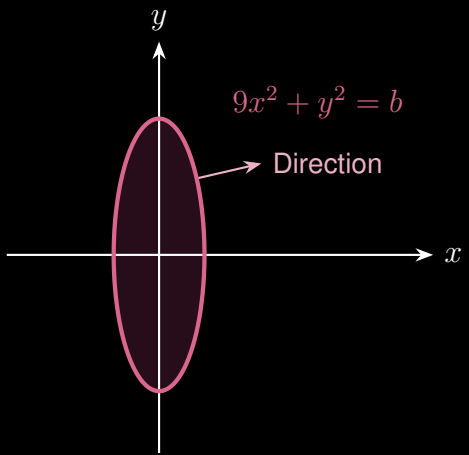
Optimal solution: $x = 0, y = 9$

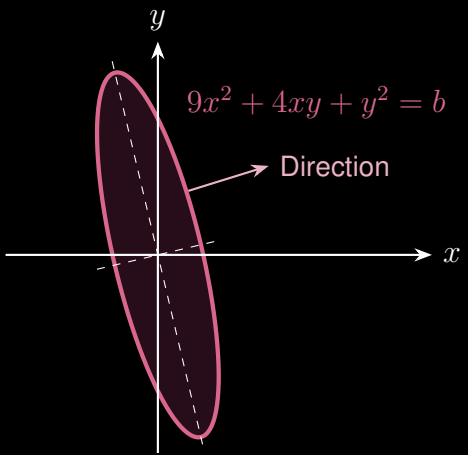


$$\max_{x,y} \; 9x^2 + y^2 \qquad \rightarrow \qquad \max_{x,y} \; 9x^2 + 4xy + y^2$$



$$\max_{x,y} \; 9x^2 + y^2 \quad \rightarrow \quad \max_{x,y} \; 9x^2 + 4xy + y^2$$



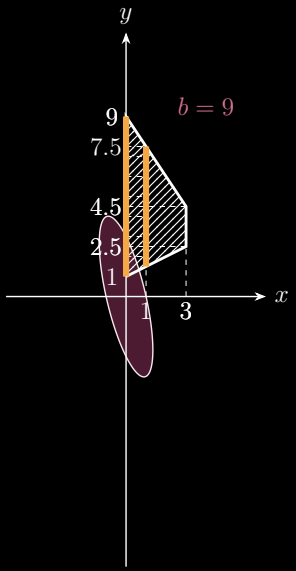


Mixed-Integer Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + 4xy + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective $9x^2 + 4xy + y^2 = b$

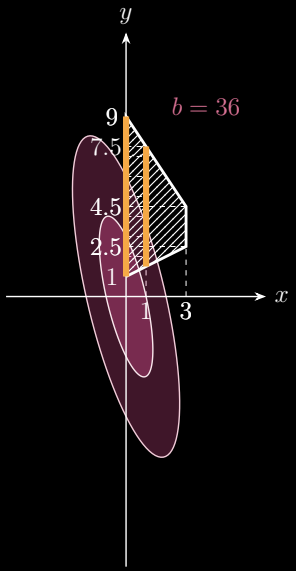


Mixed-Integer Quadratic Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + 4xy + y^2 \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Objective $9x^2 + 4xy + y^2 = b$



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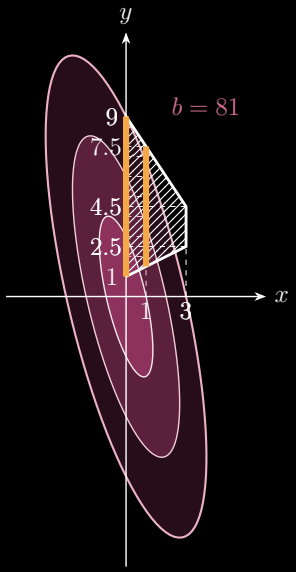
Objective $9x^2 + 4xy + y^2 = b$

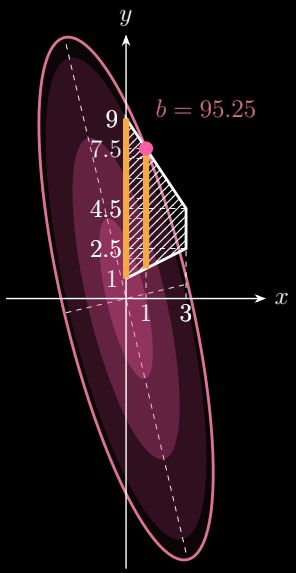


Mixed-Integer Quadratic Programming

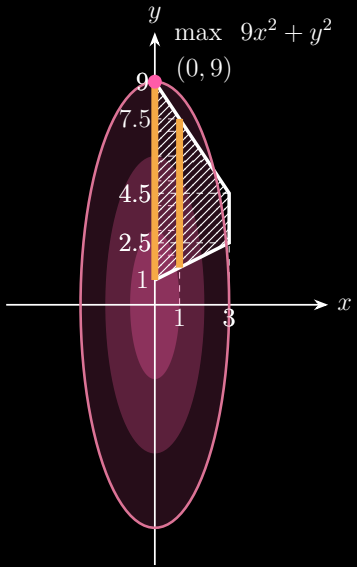
Optimization problem:

$$
\begin{aligned}
\max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 9x^2 + 4xy + y^2 \\
\text{s.t.} \quad & x, y \geq 0 \\
& 3x + 2y \leq 18 \\
& -x + 2y \geq 2 \\
& x \leq 3
\end{aligned}
$$

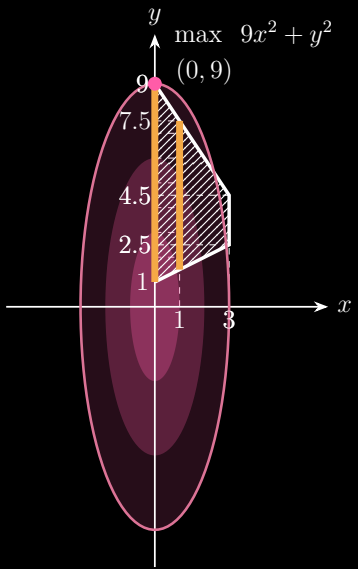
Optimal solution: $x = 1, y = 7.5$

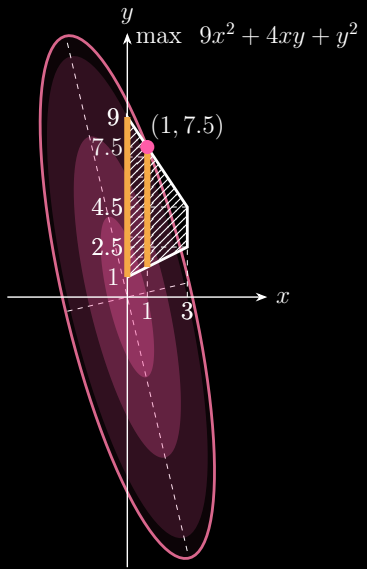


Different objectives $\rightarrow$ different optimal solutions



Different objectives $\rightarrow$ different optimal solutions





Thanks for your attention!

About me:

- Homepage: https://xinychen.github.io
- How to reach me: chenxy346@gmail.com