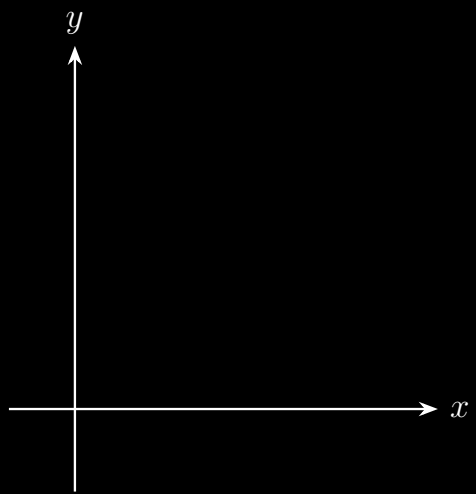
Linear Programming

Optimization problem:

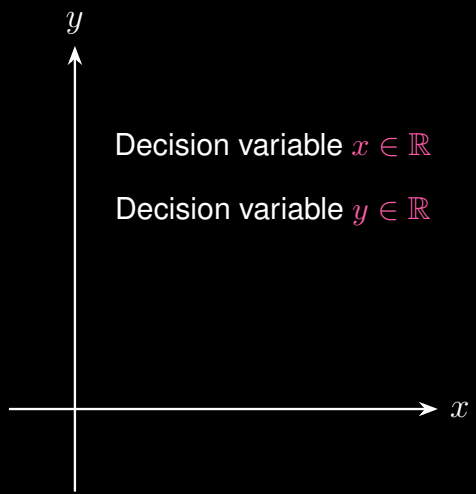
$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$



Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$



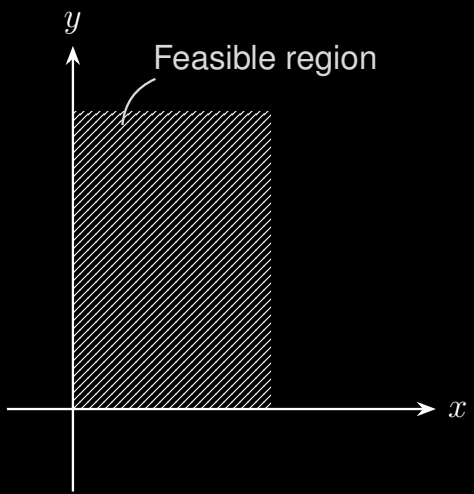
Decision variable $x \in \mathbb{R}$

Decision variable $y \in \mathbb{R}$

Linear Programming

Optimization problem:

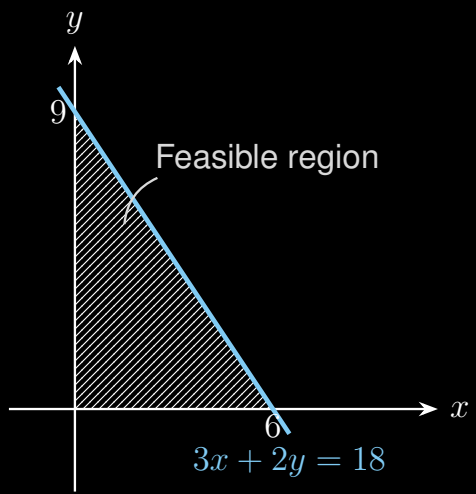
$$\begin{aligned} \max_{x \in \mathbb{R},\, y \in \mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \end{aligned}$$



Linear Programming

Optimization problem:

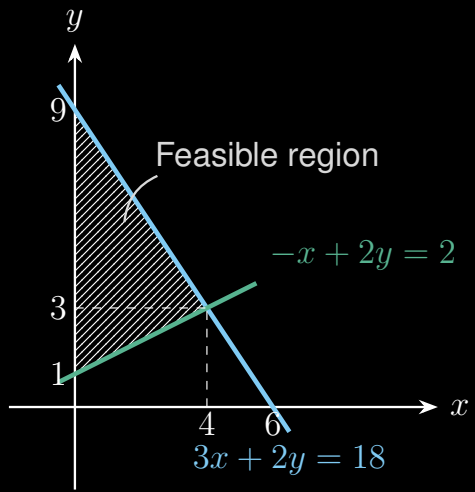
$$\begin{aligned} \max_{x \in \mathbb{R},\, y \in \mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \end{aligned}$$



Linear Programming

Optimization problem:

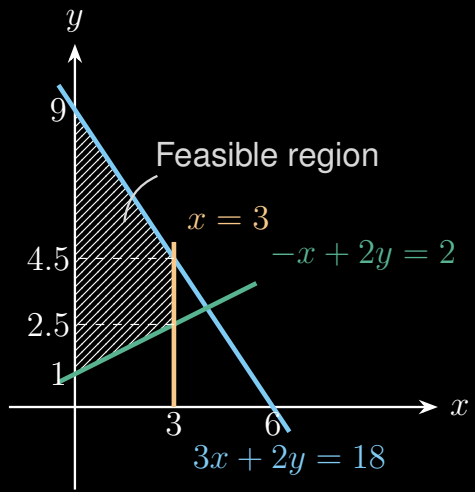
$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \end{aligned}$$



Linear Programming

Optimization problem:

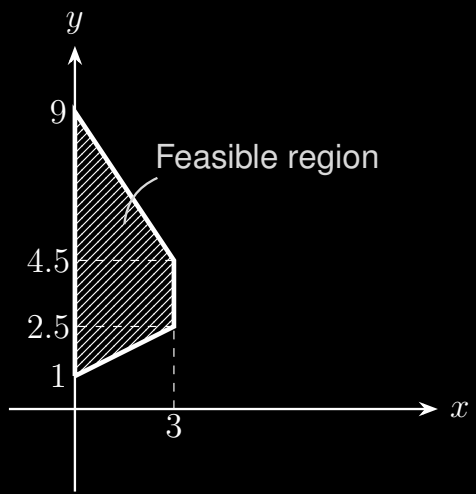
$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$



Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

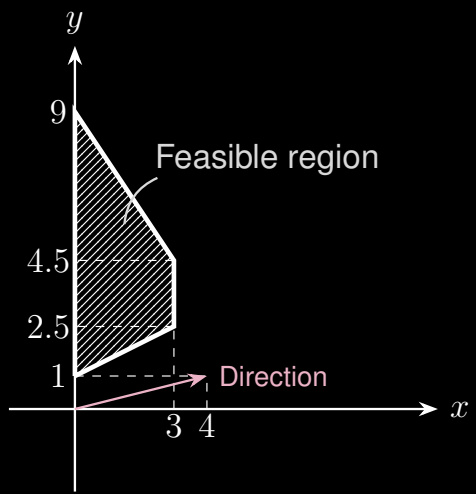


Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Direction vector $(4, 1)$

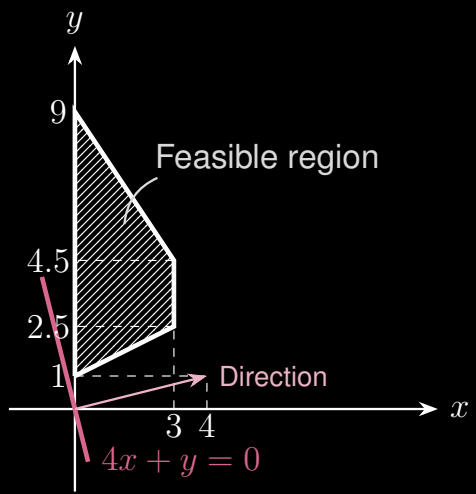


Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Direction vector $(4, 1)$

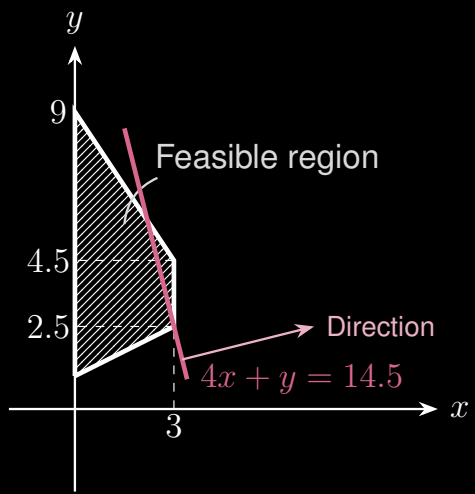


Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Direction vector $(4, 1)$

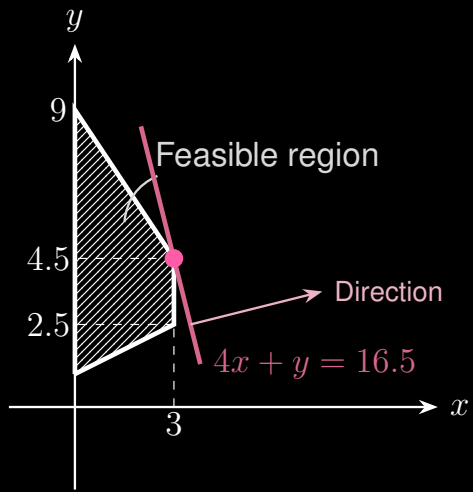


Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Optimal solution: $x = 3, y = 4.5$



How about integer decision variables?

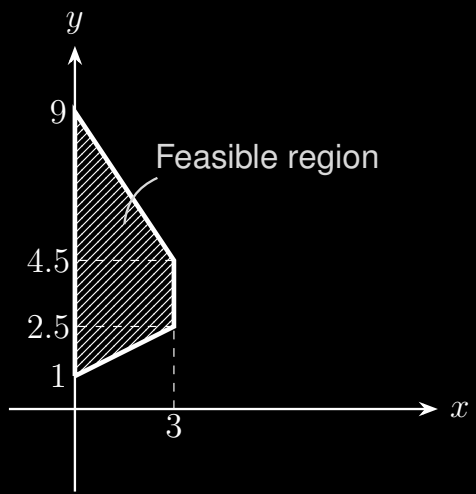
$x \in \mathbb{Z}$ and $y \in \mathbb{Z}$

Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{Z},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (●)

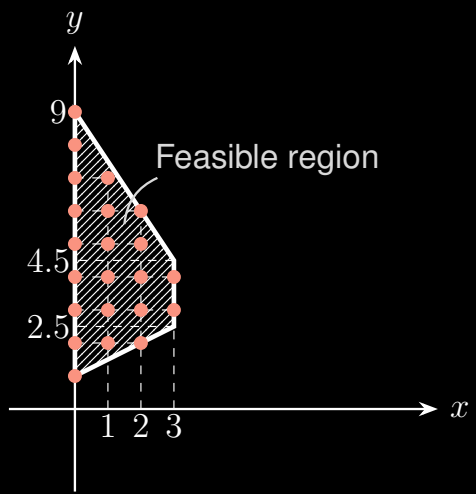


Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{Z},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (●)

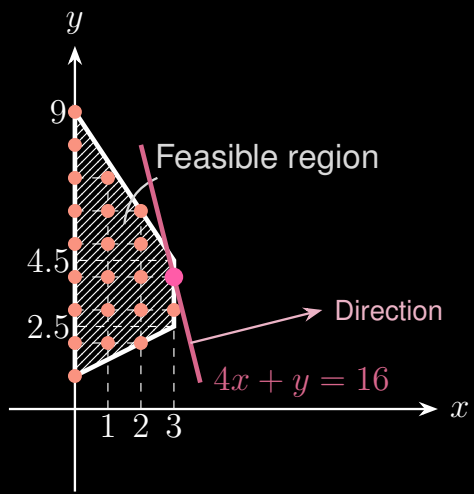


Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{Z},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (●)

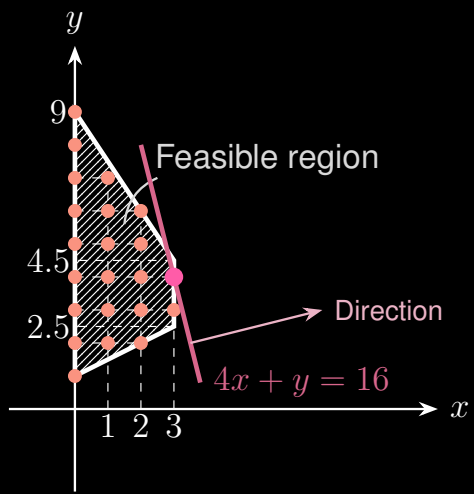


Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{Z},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Optimal solution: $x = 3, y = 4$



How about integer decision variables?

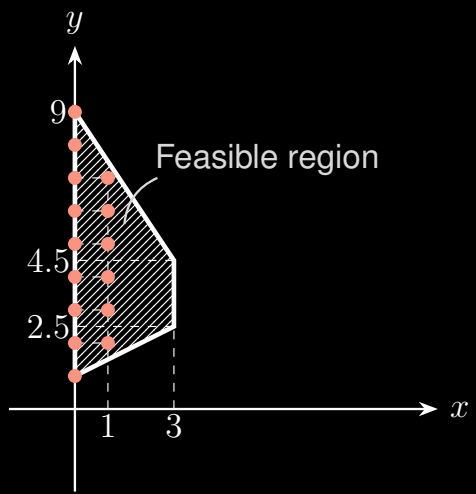
$x \in \{0, 1\}$ and $y \in \mathbb{Z}$

Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (● $x = 0, 1$)

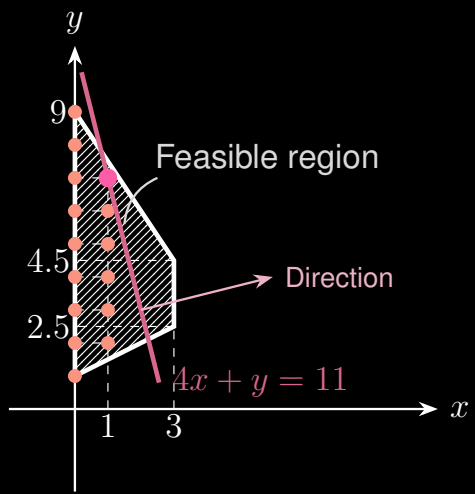


Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions ($\bullet$ $x = 0, 1$)

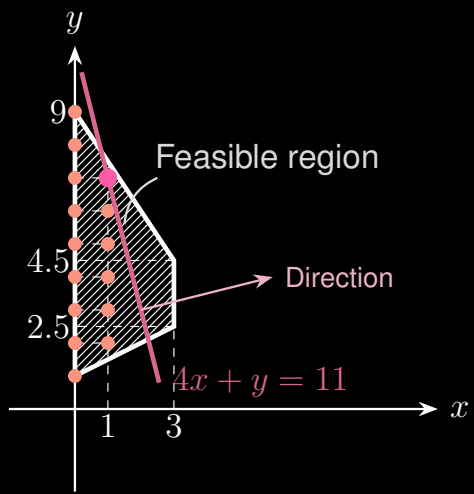


Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{Z}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Optimal solution: $x = 1, y = 7$



How about mixed-integer decision variables?

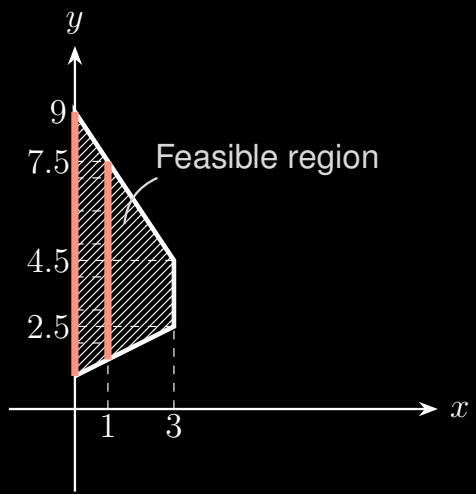
$x \in \{0, 1\}$ and $y \in \mathbb{R}$

Mixed-Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x \in \{0,1\},\, y \in \mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (—)

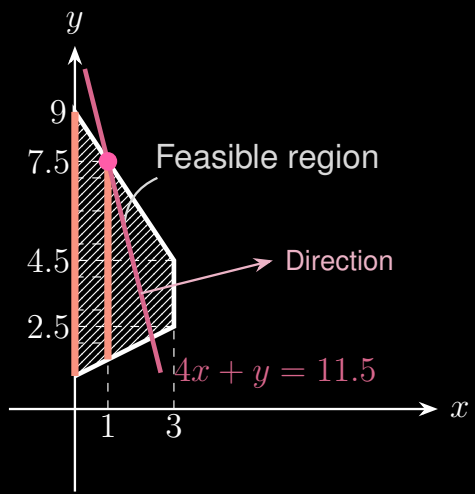


Mixed-Integer Linear Programming

Optimization problem:

$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Feasible solutions (—)

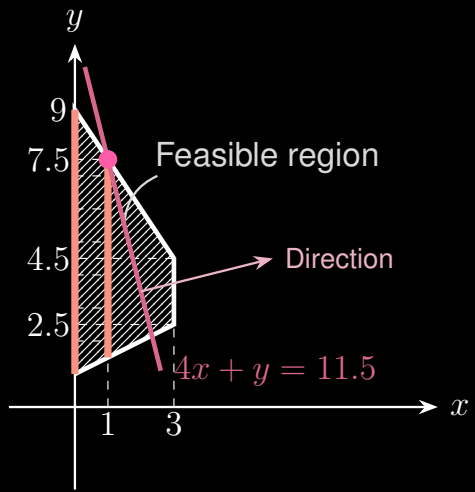


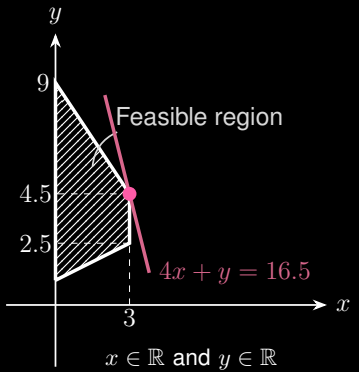
Mixed-Integer Linear Programming

Optimization problem:

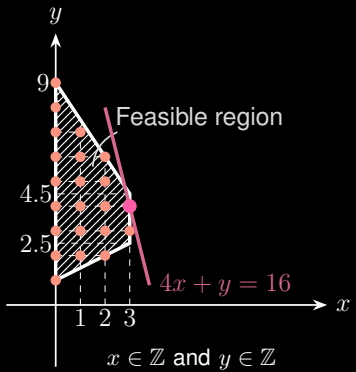
$$\begin{aligned} \max_{x\in\{0,1\},\, y\in\mathbb{R}} \quad & 4x + y \\ \text{s.t.} \quad & x, y \geq 0 \\ & 3x + 2y \leq 18 \\ & -x + 2y \geq 2 \\ & x \leq 3 \end{aligned}$$

Optimal solution: $x = 1, y = 7.5$

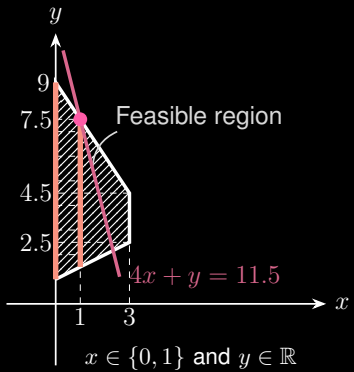




$x \in \mathbb{R}$ and $y \in \mathbb{R}$



$x \in \mathbb{Z}$ and $y \in \mathbb{Z}$



$x \in \{0, 1\}$ and $y \in \mathbb{R}$

Thanks for your attention!

About me:

- Homepage: https://xinychen.github.io
- How to reach me: chenxy346@gmail.com