POLYTECHNIQUE MONTRÉAL
UNIVERSITÉ D'INGÉNIERIE

IVADO

Spatiotemporal Traffic Data Imputation and Forecasting with Tensor Learning

Ph.D. Research Project

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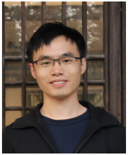
May 24, 2022



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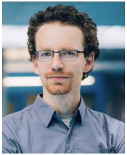

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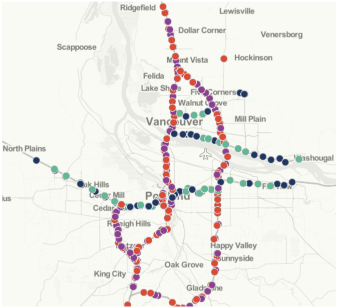
Outline

- **Motivation**
  - Multivariate traffic time series
  - Multidimensional traffic time series
  - Multiple data behaviors
- **Literature Review**
  - Spatiotemporal traffic data imputation
  - Spatiotemporal traffic forecasting
  - Low-rank tensor learning
- **Objective A**
  - Spatiotemporal traffic data imputation
- **Objective B**
  - High-dimensional traffic forecasting
  - Multidimensional traffic forecasting
- **Objective C**
  - Multivariate traffic forecasting on sparse data
  - Multidimensional traffic forecasting on sparse data
- **Conclusion**

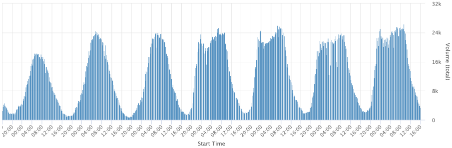
Multivariate Traffic Time Series

Many spatiotemporal traffic time series data are in the form of **matrix**.

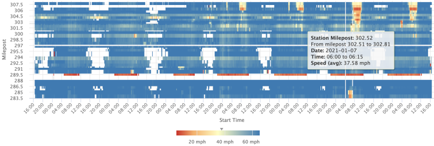
- **Example**: Portland highway traffic data[1].



Detector locations



Traffic volume



Traffic speed

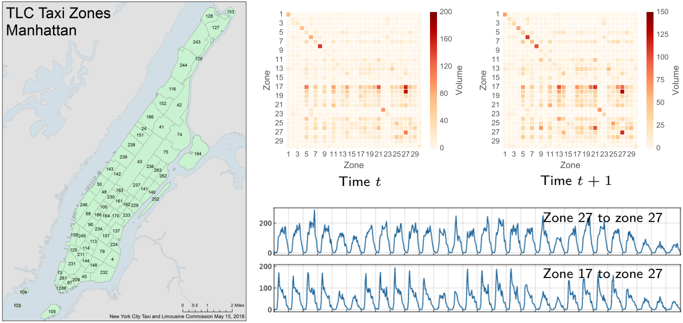
- $\boldsymbol{X} \in \mathbb{R}^{N \times T}$ with N spatial locations $\times$ T time steps

[1] https://portal.its.pdx.edu/home

Multidimensional Traffic Time Series

Many spatiotemporal traffic time series data are in the form of **tensor**.

- **Example:** NYC (hourly) taxi flow data[2].



- $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{M \times N \times T}$ with M zones $\times$ N zones $\times$ T time steps

[2] https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page

Multiple Data Behaviors

Spatiotemporal traffic data are time series, but they involve multiple data behaviors.

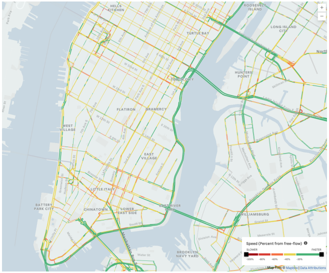
- Incompleteness & sparsity
- High-dimensionality
- Multidimensionality
- Noises & outliers
- Time-varying behavior
- Nonstationarity
- ......

In addition, spatiotemporal correlations are also very important.

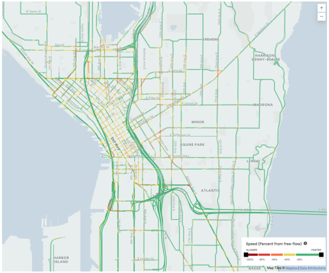
Multiple Data Behaviors

Sparsity & high-dimensionality

- Uber (hourly) movement speed data[3]



NYC movement



Seattle movement

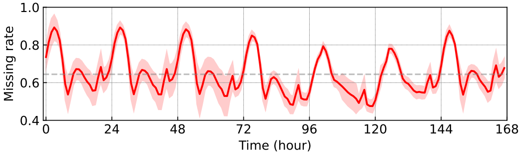
- The average speed on a given road segment for each hour of each day.
- Hourly speeds are computed when road segments have 5+ unique trips.
- **Issue**: insufficient sampling of ridesharing vehicles on the road network.

[3] https://movement.uber.com/

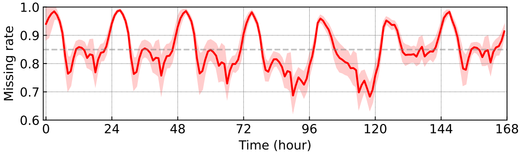
Multiple Data Behaviors

Sparsity & high-dimensionality

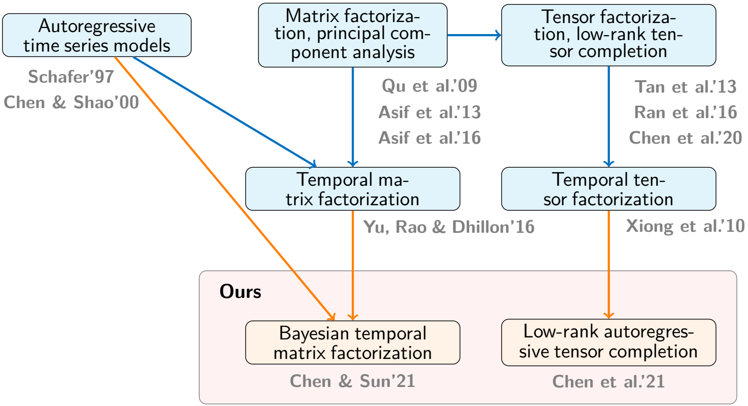
- **NYC** movement speed data (2019)
  - **98,210** road segments & 8,760 time steps (hours)
  - Overall missing rate: **64.43%**



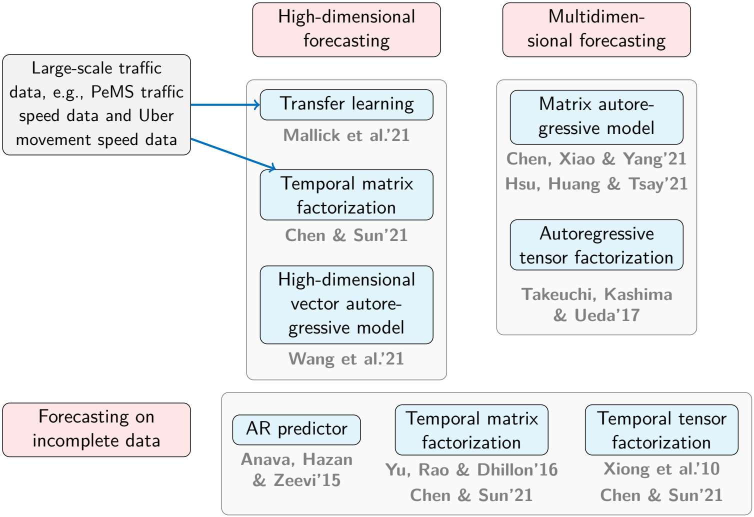
- **Seattle** movement speed data (2019)
  - **63,490** road segments & 8,760 time steps (hours)
  - Overall missing rate: **84.95%**



Spatiotemporal Traffic Data Imputation



Spatiotemporal Traffic Forecasting



Low-Rank Tensor Learning

1927 — Higher-Order SVD

Frank Lauren Hitchcock

1960s — Tucker Decomposition

Ledyard R. Tucker

1970 — CP Decomposition

J. Douglas Carroll
Jih-Jie Chang
Richard A. Harshman

2009 — Tensor Decompositions and Applications

Tamara G. Kolda

2011 — Tensor-Train Decomposition

Ivan Oseledets

Low-Rank Tensor Learning

- Low-rank matrix/tensor completion

Candès & Recht'09: Convex nuclear norm minimization for matrix completion.

$$\min_{\boldsymbol{X}} \|\boldsymbol{X}\|_* \quad \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{X}) = \mathcal{P}_\Omega(\boldsymbol{Y})$$

Cai, Candès & Shen'10: Singular value thresholding algorithm.

$$\begin{cases} \boldsymbol{X}^\ell = \mathcal{D}_\tau\left(\boldsymbol{Z}^{\ell-1}\right) \\ \boldsymbol{Z}^\ell = \boldsymbol{Z}^{\ell-1} + \delta_\ell \mathcal{P}_\Omega\left(\boldsymbol{Y} - \boldsymbol{X}^\ell\right) \end{cases}$$

Zhang et al.'12: Nonconvex truncated nuclear norm minimization.

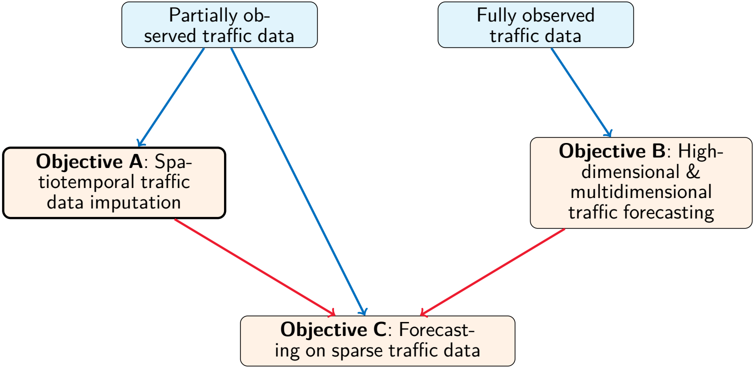
Liu et al.'13: Convex nuclear norm minimization for tensor completion.

$$\min_{\boldsymbol{\mathcal{X}}} \|\boldsymbol{\mathcal{X}}\|_* \quad \text{s.t. } \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{Y}})$$

Lu, Peng & Wei'19: Tensor nuclear norm induced by linear transform.

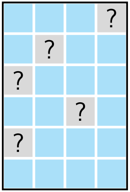
A Whole Picture of Objectives

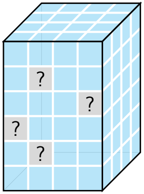
We are working on **spatiotemporal traffic data modeling**.



Spatiotemporal Traffic Data Imputation

- **Objective A:** Given a multivariate time series data like $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ or a multidimensional time series data like $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$, impute the missing values of the data.





[Q]

- How to reconstruct missing values from observed data?
- How to make use of spatiotemporal correlations?
- How to make use of traffic time series dynamics?

Spatiotemporal Traffic Data Imputation

Low-rank matrix completion (Candès & Recht'09)

For any partially observed data matrix $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$ with observed index set Ω, then low-rank matrix completion takes the form of

$$\begin{aligned} \min_{\boldsymbol{X}} \; & \|\boldsymbol{X}\|_* \\ \text{s.t. } & \mathcal{P}_{\Omega}(\boldsymbol{X}) = \mathcal{P}_{\Omega}(\boldsymbol{Y}). \end{aligned} \tag{1}$$

Low-rank tensor completion (Liu et al.'13)

For any partially observed data matrix $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$ with observed index set Ω, then low-rank matrix completion takes the form of

$$\begin{aligned} \min_{\boldsymbol{\mathcal{X}}} \; & \|\boldsymbol{\mathcal{X}}\|_* \\ \text{s.t. } & \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{X}}) = \mathcal{P}_{\Omega}(\boldsymbol{\mathcal{Y}}). \end{aligned} \tag{2}$$

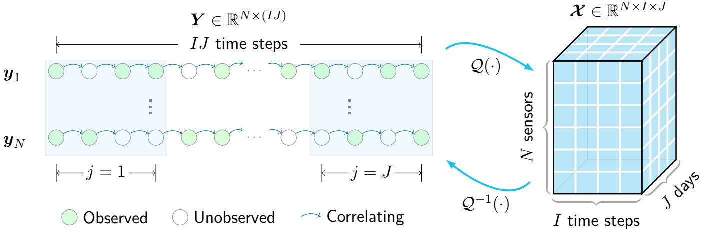
- **Limitation:** Only cover the global consistency.
- **Comment:** For modeling spatiotemporal traffic data, local consistency (e.g., temporal correlations) is also important.

Spatiotemporal Traffic Data Imputation

Low-rank autoregressive tensor completion (LATC)

$$\min_{\boldsymbol{\mathcal{X}}} \ \|\boldsymbol{\mathcal{X}}\|_* + \frac{\lambda}{2}\sum_{n=1}^{N}\sum_{t=d+1}^{T}\Big(z_{n,t} - \sum_{k=1}^{d} a_{n,k} z_{n,t-k}\Big)^2$$
$$\text{s.t.} \ \begin{cases} \boldsymbol{\mathcal{X}} = \mathcal{Q}(\boldsymbol{Z}), \\ \mathcal{P}_{\Omega}(\boldsymbol{Z}) = \mathcal{P}_{\Omega}(\boldsymbol{Y}). \end{cases} \quad (3)$$

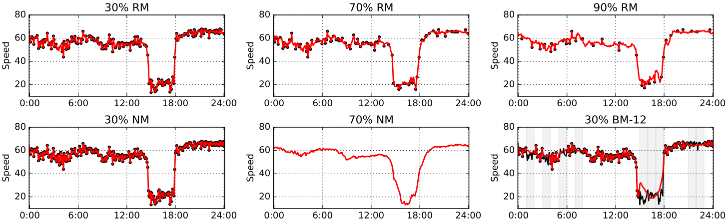
- **Advantage:** Global consistency + local consistency.



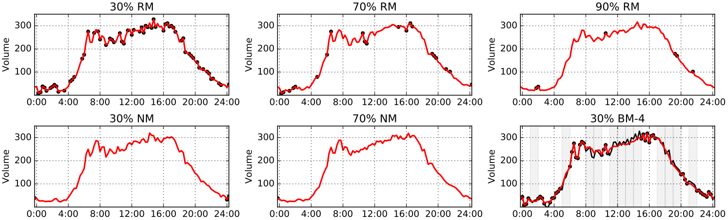
Spatiotemporal Traffic Data Imputation

LATC imputation

- Seattle freeway traffic speed data



- Portland highway traffic volume data



A Whole Picture of Objectives

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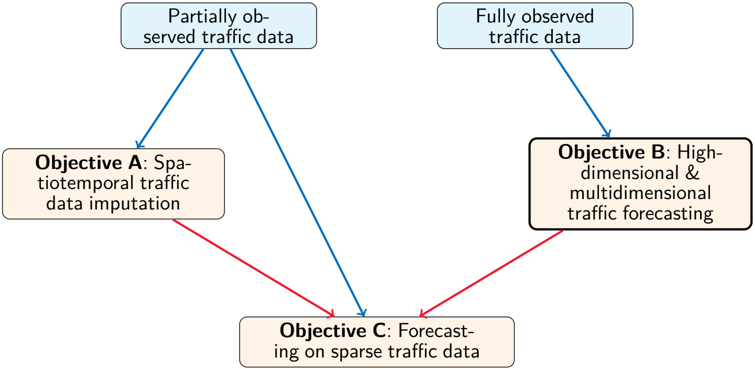
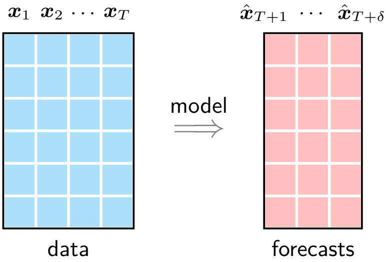


High-Dimensional Traffic Forecasting

- **Objective B-1:** Given a multivariate traffic time series $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T \in \mathbb{R}^N$ with $N \gg T$ ("tall-skinny"), forecast data points $\boldsymbol{x}_{T+\delta}, \delta \in \mathbb{N}^+$.

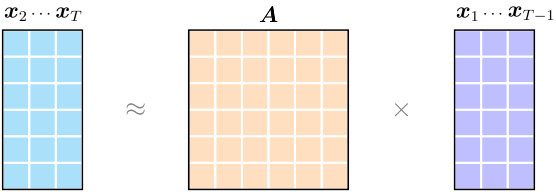


- **Solution:** For time series $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T \in \mathbb{R}^N$, the dth-order vector autoregressive (VAR(d)) model: $\boldsymbol{x}_t = \sum_{k=1}^{d} \boldsymbol{A}_k \boldsymbol{x}_{t-k} + \boldsymbol{\epsilon}_t$.
- **Advantage:** Co-evolution patterns
- **Limitation:** Over-parameterization

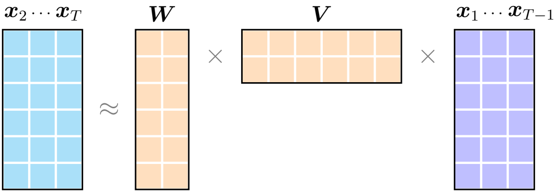
High-Dimensional Traffic Forecasting

VAR(1) model

- Over-parameterization in the case of $N \gg T$.



- Reduced-rank autoregression: $\boldsymbol{A} = \boldsymbol{W}\boldsymbol{V}$ with $\boldsymbol{W} \in \mathbb{R}^{N \times R}, \boldsymbol{V} \in \mathbb{R}^{R \times N}$.



High-Dimensional Traffic Forecasting

VAR(d) model

- Recall that $\boldsymbol{x}_t = \sum_{k=1}^{d} \boldsymbol{A}_k \boldsymbol{x}_{t-k} + \boldsymbol{\epsilon}_t$.
- Coefficients $\boldsymbol{A}_k \in \mathbb{R}^{N \times N}, k = 1, \ldots, d$ are tensor, e.g., $\boldsymbol{\mathcal{A}} \in \mathbb{R}^{N \times N \times d}$.

VAR(d) with Tucker decomposition (Wang et al.'21)

For VAR(d) on the multivariate time series $\boldsymbol{x}_t \in \mathbb{R}^N, t = 1, \ldots, T$, the reduced-rank VAR via Tucker decomposition is given by

$$\min_{\boldsymbol{\mathcal{G}}, \boldsymbol{W}, \boldsymbol{U}, \boldsymbol{H}} \frac{1}{2} \sum_{t=d+1}^{T} \|\boldsymbol{x}_t - (\boldsymbol{\mathcal{G}} \times_1 \boldsymbol{W} \times_2 \boldsymbol{U} \times_3 \boldsymbol{H})_{(1)} \boldsymbol{z}_t\|_2^2 \tag{4}$$

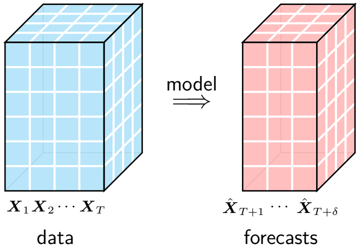
where $\boldsymbol{z}_t = (\boldsymbol{x}_{t-1}^\top, \cdots, \boldsymbol{x}_{t-d}^\top)^\top \in \mathbb{R}^{dN}$. The multilinear rank is (R_1, R_2, R_3). $\boldsymbol{\mathcal{G}} \in \mathbb{R}^{R_1 \times R_2 \times R_3}$ is the core tensor, while $\boldsymbol{W} \in \mathbb{R}^{N \times R_1}$, $\boldsymbol{U} \in \mathbb{R}^{N \times R_2}$, and $\boldsymbol{H} \in \mathbb{R}^{d \times R_3}$ are the component matrices.

Advantage: High compression rate.

Limitations: Nonconvex optimization; the model is failing in nonstationary time series.

Multidimensional Traffic Forecasting

- **Objective B-2:** Given a multidimensional traffic time series $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_T \in \mathbb{R}^{M \times N}$, forecast data points $\boldsymbol{X}_{T+\delta}, \delta \in \mathbb{N}^+$.



[Q]

- How to perform forecasting on this kind of data?
- How to preserve the intrinsic tensor representation of data?

Multidimensional Traffic Forecasting

Matrix autoregressive model (Chen, Xiao & Yang'21)

Given matrix-variate time series $\boldsymbol{X}_t \in \mathbb{R}^{M \times N}, t = 1, \ldots, T$, then the dth-order matrix autoregressive (MAR(d)) model takes the form of

$$\boldsymbol{X}_t = \sum_{k=1}^{d} \boldsymbol{A}_k \boldsymbol{X}_{t-k} \boldsymbol{B}_k^\top + \boldsymbol{E}_t \tag{5}$$

where $\boldsymbol{A}_k \in \mathbb{R}^{M \times M}, \boldsymbol{B}_k \in \mathbb{R}^{N \times N}, k = 1, \ldots, d$ are the coefficient matrices.

Advantages:

- Preserve the intrinsic tensor representation.
- Reduce parameters in autoregressive models (if $n = \max\{M, N\}$), e.g.,

$$\mathcal{O}(n^4) \text{ in VAR}(1) \quad \text{vs.} \quad \mathcal{O}(n^2) \text{ in MAR}(1)$$

Limitation: Failing in nonstationary time series.

A Whole Picture of Objectives

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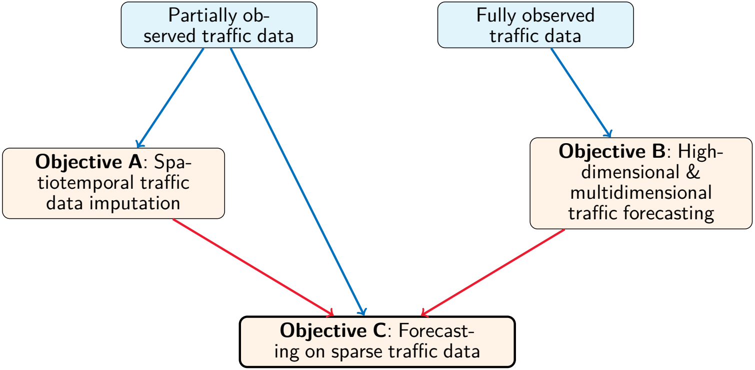
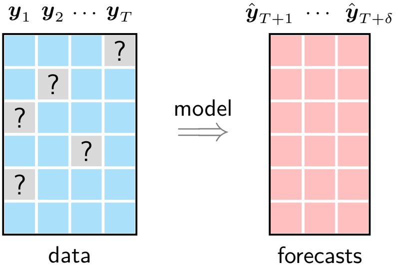


Multivariate Traffic Forecasting on Sparse Data

- **Objective C-1:** Given a partially observed data $\boldsymbol{Y} \in \mathbb{R}^{N\times T}$ consisting of time series $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_T \in \mathbb{R}^N$, forecast data points $\boldsymbol{y}_{T+\delta}, \delta \in \mathbb{N}^+$.



[Q]

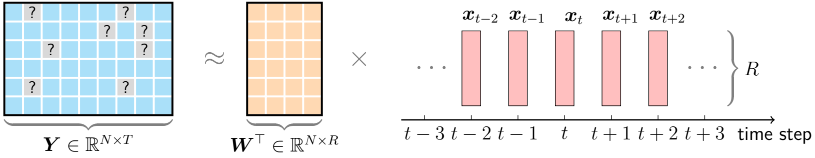
- How to learn from *high-dimensional* and *sparse* data?
- How to model *nonstationarity* in time series?
- How to perform forecasting on these time series?

Multivariate Traffic Forecasting on Sparse Data

Temporal matrix factorization (Yu et al.'16; Chen & Sun'21)

Given any partially observed time series data $\boldsymbol{Y}\in\mathbb{R}^{N\times T}$ with observed index set Ω, then temporal matrix factorization assumes a dth-order vector autoregressive (VAR) process on the temporal factor matrix:

$$\min_{\boldsymbol{W},\boldsymbol{X},\{\boldsymbol{A}_k\}_{k=1}^{d}} \frac{1}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})\|_F^2+\frac{\rho}{2}(\|\boldsymbol{W}\|_F^2+\|\boldsymbol{X}\|_F^2) + \frac{\lambda}{2}\sum_{t=d+1}^{T}\|\boldsymbol{x}_t-\sum_{k=1}^{d}\boldsymbol{A}_k\boldsymbol{x}_{t-k}\|_2^2 \tag{6}$$



🙁 VAR is usually built on stationary time series (temporal factors).

Multivariate Traffic Forecasting on Sparse Data

Nonstationary temporal matrix factorization (NoTMF)

Given any partially observed time series data $\boldsymbol{Y}\in\mathbb{R}^{N\times T}$ with observed index set Ω, then we assume a season-m differencing on the latent temporal factors:

$$\begin{aligned}\min_{\boldsymbol{W},\boldsymbol{X},\{\boldsymbol{A}_k\}_{k=1}^{d}}\quad&\frac{1}{2}\|\mathcal{P}_{\Omega}(\boldsymbol{Y}-\boldsymbol{W}^\top\boldsymbol{X})\|_F^2+\frac{\rho}{2}(\|\boldsymbol{W}\|_F^2+\|\boldsymbol{X}\|_F^2)\\&+\frac{\lambda}{2}\sum_{t=d+m+1}^{T}\|(\boldsymbol{x}_t-\boldsymbol{x}_{t-m})-\sum_{k=1}^{d}\boldsymbol{A}_k(\boldsymbol{x}_{t-k}-\boldsymbol{x}_{t-m-k})\|_2^2\end{aligned}\tag{7}$$

- First-order differencing $\boldsymbol{x}_t'=\boldsymbol{x}_t-\boldsymbol{x}_{t-1}$.
- Second-order differencing $\boldsymbol{x}_t''=(\boldsymbol{x}_t-\boldsymbol{x}_{t-1})-(\boldsymbol{x}_{t-1}-\boldsymbol{x}_{t-2})$.
- Twice-differenced series $\boldsymbol{x}_t''=(\boldsymbol{x}_t-\boldsymbol{x}_{t-m})-(\boldsymbol{x}_{t-1}-\boldsymbol{x}_{t-m-1})$.

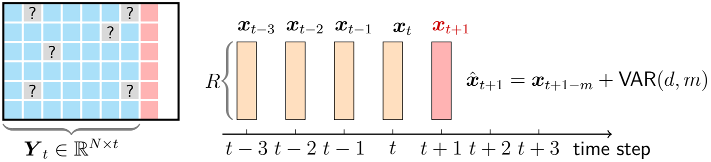
🙂 Stationarizing a time series with differencing can improve the prediction.[4]

[4] Stationarity and differencing: https://otexts.com/fpp2/stationarity.html

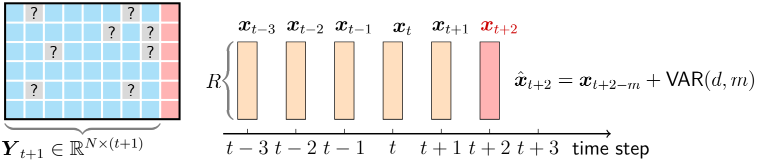
Multivariate Traffic Forecasting on Sparse Data

NoTMF forecasting on streaming data?

- NoTMF: Use $\boldsymbol{Y}_t$ to estimate $\{\boldsymbol{W}, \boldsymbol{X}, \boldsymbol{A}\}$.



- Online forecasting (Gultekin & Paisley'18): Fix $\boldsymbol{W}$ and use $\boldsymbol{Y}_{t+1}$ to update $\{\boldsymbol{X}, \boldsymbol{A}\}$.

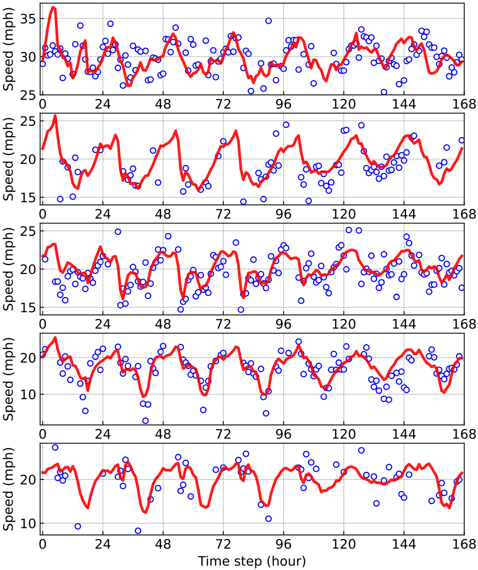


Multivariate Traffic Forecasting on Sparse Data

Forecasting performance on NYC Uber movement speed data (MAPE/RMSE):

- NoTMF outperforms other models.
- Seasonal differencing in NoTMF is important.

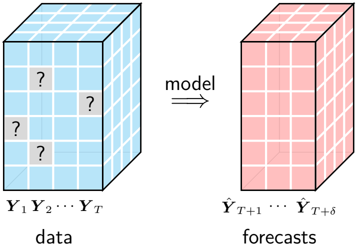
δ	d	NoTMF ($m=24$)	NoTMF ($m=168$)	NoTMF-twice ($m=168$)	TMF	TRMF	BTMF
1	1	13.63/2.88	13.53/2.86	**13.45**/**2.85**	13.74/2.90	14.50/3.12	14.94/3.13
	2	13.47/**2.84**	**13.41**/**2.84**	13.42/**2.84**	13.53/2.85	14.14/3.05	15.70/3.41
	3	13.46/2.84	**13.39**/**2.83**	13.43/2.84	13.47/**2.83**	13.87/2.96	15.80/3.34
	6	13.41/**2.83**	**13.39**/**2.83**	13.41/**2.83**	13.40/**2.83**	14.00/2.98	15.45/3.27
2	1	13.91/2.96	13.76/2.94	**13.70**/**2.92**	14.24/3.00	15.85/3.43	15.33/3.21
	2	13.77/2.92	**13.63**/**2.89**	13.72/2.92	13.87/2.91	15.04/3.31	15.87/3.32
	3	13.72/2.91	**13.61**/**2.89**	13.73/2.92	13.81/**2.89**	15.25/3.36	15.69/3.33
	6	13.59/2.87	**13.57**/2.88	13.68/2.91	13.63/**2.86**	14.92/3.24	15.91/3.39
3	1	14.30/3.05	14.06/3.02	**14.02**/**3.00**	14.81/3.12	17.52/3.83	15.86/3.32
	2	14.01/2.98	**13.84**/**2.94**	13.96/2.98	14.26/2.98	17.32/4.00	16.30/3.40
	3	13.95/2.97	**13.79**/**2.93**	13.98/2.98	14.04/2.94	16.91/3.71	16.56/3.49
	6	13.78/**2.92**	**13.73**/**2.92**	13.91/2.96	13.94/**2.92**	16.72/3.65	15.49/3.27
6	1	**14.61**/**3.11**	14.67/3.20	14.98/3.32	15.41/3.21	21.20/4.70	15.99/3.32
	2	**14.30**/**3.03**	14.33/3.09	14.90/3.28	14.85/3.07	20.87/5.01	16.04/3.33
	3	**14.26**/3.03	14.28/3.09	14.86/3.26	14.57/**3.01**	20.08/4.65	15.67/3.28
	6	**14.06**/**2.97**	14.16/3.06	14.80/3.23	14.47/3.00	20.40/4.35	16.38/3.50



Speed (mph)
Time step (hour)
0
24
48
72
96
120
144
168

Multidimensional Traffic Forecasting on Sparse Data

- **Objective C-2:** Given a partially observed data $\boldsymbol{\mathcal{Y}} \in \mathbb{R}^{M \times N \times T}$ consisting of time series $\boldsymbol{Y}_1, \ldots, \boldsymbol{Y}_T \in \mathbb{R}^{M \times N}$, forecast data points $\boldsymbol{Y}_{T+\delta}, \delta \in \mathbb{N}^+$.



- **Solution:** Temporal tensor factorization, e.g., tensor factorization + VAR (on latent temporal factors).

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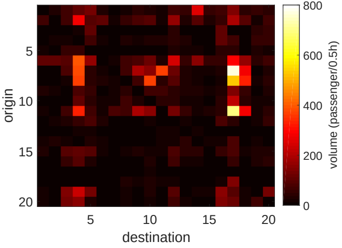
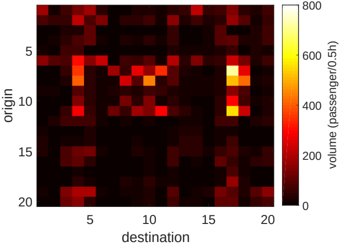
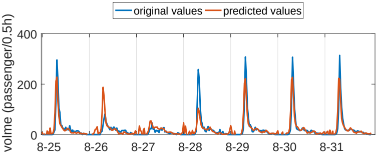


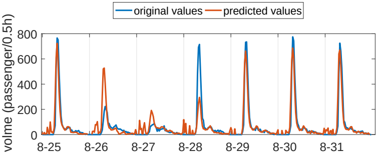
Original OD volume matrix.



Predicted OD volume matrix.



Volume of the #(6,17) OD pair.



Volume of the #(7,17) OD pair.

Conclusion

- **Contributions**
  - *Objective A: Spatiotemporal traffic data imputation.* Develop a low-rank temporal modeling framework and improve the imputation accuracy, efficiency, and scalability.
  - *Objective B: High-dimensional and multidimensional forecasting.* Fast and accurate forecasting approach for high-dimensional and large-scale data; tensor representation based autoregressive model for multidimensional data.
  - *Objective C: Forecasting on sparse data.* Low-rank temporal modeling framework for traffic time series forecasting in the presence of missing values.
- **Significance**
  - Improve traffic data quality.
  - Support data-driven intelligent transportation applications.

Research Work during Ph.D. Research

- **Publications**
  - [**J1**] **X. Chen**, M. Lei, N. Saunier, L. Sun (2021). Low-rank autoregressive tensor completion for spatiotemporal traffic data imputation. *IEEE Transactions on Intelligent Transportation Systems*. (Early access)
  - [**J2**] **X. Chen**, Y. Chen, N. Saunier, L. Sun (2021). Scalable low-rank tensor learning for spatiotemporal traffic data imputation. *Transportation Research Part C: Emerging Technologies*, 129: 103226.
  - [**C1**] **X. Chen**, M. Lei, N. Saunier, L. Sun (2021). Low-rank autoregressive tensor completion for spatiotemporal traffic data imputation. *The 7th SIGKDD Workshop on Mining and Learning from Time Series (MiLeTS)* at KDD 2021.

- **Preprint** (under review)
  - [**P1**] **X. Chen**, C. Zhang, X.L. Zhao, N. Saunier, L. Sun (2022). Nonstationary temporal matrix factorization for multivariate time series forecasting. arXiv preprint arXiv:2203.10651.

- **Open-source projects**
  - **transdim**: Machine learning for spatiotemporal traffic data imputation and forecasting. (780 stars & 240 forks on GitHub)
    https://github.com/xinychen/transdim
  - **tracebase**: Multivariate time series forecasting on high-dimensional and sparse Uber movement speed data. (20 stars & 5 forks on GitHub)
    https://github.com/xinychen/tracebase

Thanks for your attention!

Any Questions?

About me:

🏛 Homepage: https://xinychen.github.io

🧑‍💻 GitHub: https://github.com/xinychen (**2.4K+ stars**)

✍ Blog: https://medium.com/@xinyu.chen (**30K+ views**)

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