Orthogonal Procrustes Problem (OPP)

For any $\boldsymbol{Q} \in \mathbb{R}^{m \times r}$, the solution to

$$\min_{\boldsymbol{F}} \|\boldsymbol{F} - \boldsymbol{Q}\|_F^2$$
$$\text{s.t. } \underbrace{\boldsymbol{F}^\top \boldsymbol{F} = \boldsymbol{I}_r}_{\text{orthogonal}}$$

is

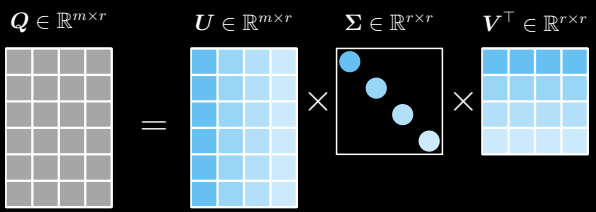
$$\boldsymbol{F} := \boldsymbol{U}\boldsymbol{V}^\top$$

Python implementation:

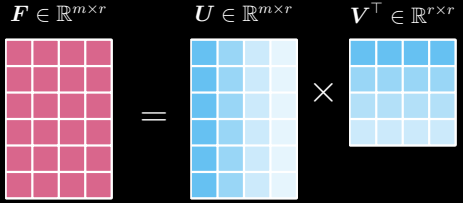
```python
import numpy as np
from numpy.linalg import svd

def opp(Q):
    U, Sigma, V = svd(Q, full_matrices = 0)
    return U @ V
```

① Singular value decomposition on $\boldsymbol{Q} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^\top$



② Matrix multiplication between $\boldsymbol{U}$ and $\boldsymbol{V}$



Thanks for your attention!

Any Questions?

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