Quantifying Time Series Periodicity with Interpretable Machine Learning

Climate Systems & Urban Human Mobility

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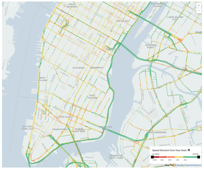
Cambridge, USA

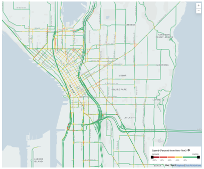
Spatiotemporal Time Series Data

- Transport & mobility & climate application scenarios

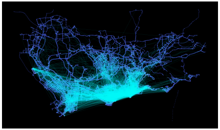
Highway (Portland)



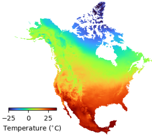
Uber movement (NYC)



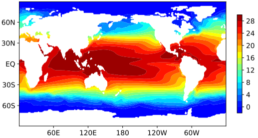
Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Temperature (NA)

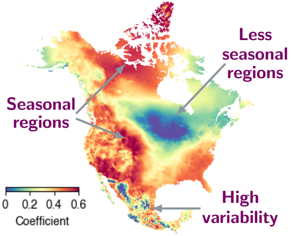


Temperature (sea surface)

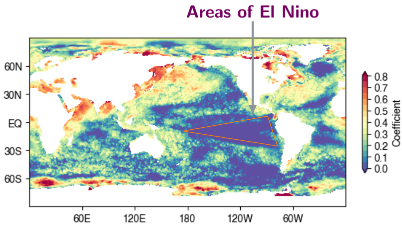
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

Motivation

Yearly temperature **seasonality** patterns in 2010s



North America



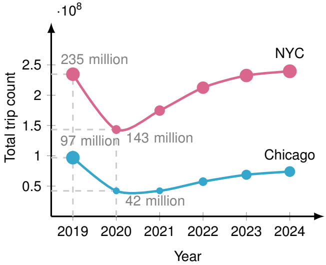
Global sea surface

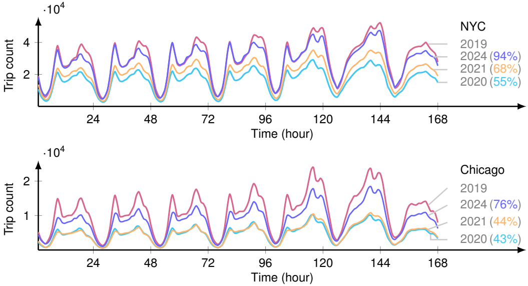
What motivate us most about **periodicity**?

1. **Monitoring climate systems**: Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).
2. **Discovering spatiotemporal patterns**: Identify periodicity pattern shift and special climate events.

Motivation

Ridesharing trip data



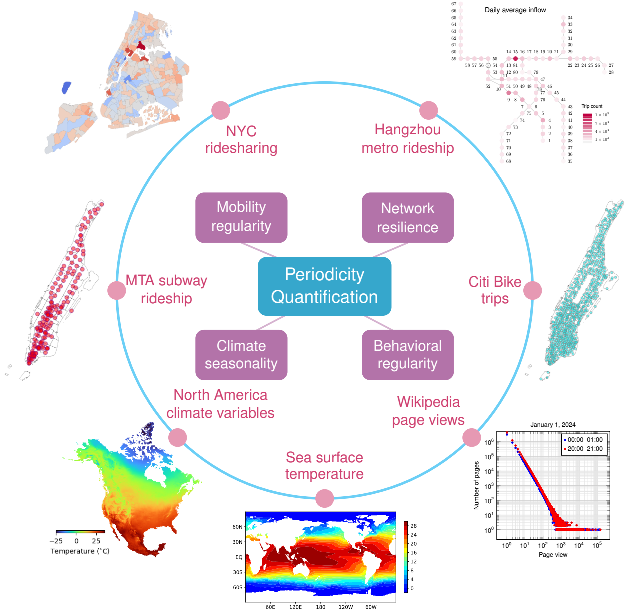


What motivate us most about **periodicity**?

❶ **Resilience and stability of systems**: Empirically measure the periodicity and predictability of urban systems.

❷ **Optimization of transport systems**: Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.

❸ **Design of sustainable transport & infrastructure**: Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.


Daily average inflow
Trip count
NYC
ridesharing
Hangzhou
metro rideship
Mobility
regularity
Network
resilience
MTA subway
rideship
Periodicity
Quantification
Citi Bike
trips
Climate
seasonality
Behavioral
regularity
North America
climate variables
Wikipedia
page views
January 1, 2024
00:00–01:00
20:00–21:00
Number of pages
Page view
Sea surface
temperature
Temperature (°C)
60N
30N
EQ
30S
60S
60E
120E
180
120W
60W

Interpretable Time Series Autoregression

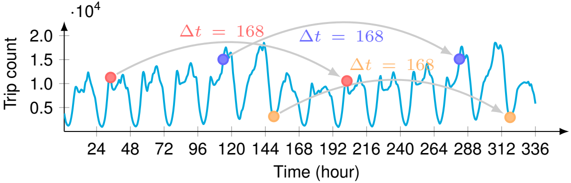
https://github.com/xinychen/integers

- Interpretable ML
- ℓ_0-norm optimization
- Climate system seasonality
- Sparse autoregression
- Mixed-integer programming
- Human mobility regularity

Valorizing Autoregression

- Time series autoregression on $\boldsymbol{x} \in \mathbb{R}^{T}$ with order $d \in \mathbb{Z}^{+}$

$$\boldsymbol{w} := \arg\min_{\boldsymbol{w}} \sum_{t=d+1}^{T} \left(x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2$$



Periodicity of hourly rideshare trip time series

- **Sparse** coefficient vector $\mapsto$ **Interpretability**?

$$\underbrace{\boldsymbol{w}}_{\text{sparsity } \|\boldsymbol{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \cdots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Sparse Autoregression

- Identify the dominant auto-correlations
  - $\tau \in \mathbb{Z}^{+}$: Upper bound of the number of nonzero entries in $\boldsymbol{w} \in \mathbb{R}^{d}$

$$\underbrace{\begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{bmatrix}}_{\tilde{\boldsymbol{x}}} \approx \underbrace{\begin{bmatrix} x_4 & x_3 & x_2 & x_1 \\ x_5 & x_4 & x_3 & x_2 \\ x_6 & x_5 & x_4 & x_3 \\ x_7 & x_6 & x_5 & x_4 \\ x_8 & x_7 & x_6 & x_5 \end{bmatrix}}_{\boldsymbol{A}} \times \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}}_{\boldsymbol{w}} \Big\} \|\boldsymbol{w}\|_0 \leq \tau$$

$$\begin{aligned} \boldsymbol{w} &:= \underset{\|\boldsymbol{w}\|_0 \leq \tau}{\arg\min} \sum_{t=d+1}^{T} \left(x_t - \sum_{k=1}^{d} w_k x_{t-k} \right)^2 \\ &= \underset{\|\boldsymbol{w}\|_0 \leq \tau}{\arg\min} \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \end{aligned}$$

- ℓ_0-norm optimization is NP-hard
- Formulate it as a mixed-integer programming
  - Introduce binary decision variables $\boldsymbol{\beta} \in \{0, 1\}^{d}$

$$\begin{aligned} &\min_{\boldsymbol{w}} \ \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ &\text{s.t.} \ \underbrace{\|\boldsymbol{w}\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{w}} \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} &\min_{\boldsymbol{w},\, \boldsymbol{\beta}} \ \|\tilde{\boldsymbol{x}} - \boldsymbol{A}\boldsymbol{w}\|_2^2 \\ &\text{s.t.} \ \underbrace{-\boldsymbol{\beta} \leq \boldsymbol{w} \leq \boldsymbol{\beta},}_{\text{bounds being either } 0 \text{ or } \pm 1} \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \boldsymbol{\beta}} \end{aligned}$$

Sparse Autoregression Done Right

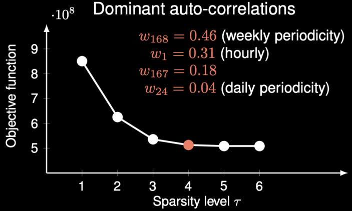
$$\min_{\boldsymbol{w},\,\boldsymbol{\beta}} \underbrace{\sum_{t=d+1}^{T}\left(x_t-\sum_{k=1}^{d}w_kx_{t-k}\right)^2}_{\text{Time series autoregression}} \quad \text{s.t. } \underbrace{-\beta_k\leq w_k\leq\beta_k}_{\text{Lower and upper bounds}},\ \underbrace{\sum_{k=1}^{d}\beta_k\leq\tau}_{\clubsuit\ \text{Sparsity}},\ \underbrace{\beta_k\in\{0,1\}}_{\text{Binary variable}}$$

- $\boldsymbol{w}\in\mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta}\in\{0,1\}^d$: Sparsity pattern
- $d=168$: Autoregression order


```python
import numpy as np
from docplex.mp.model import Model

def sparse_ar(x, d, tau):
    model = Model('Sparse Autoregression')
    T = x.shape[0]
    w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
    beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
    model.minimize(model.sum((x[t] - model.sum(w[k] * x[t - k - 1]
                                          for k in range(d))) ** 2
                             for t in range(d, T)))
    model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
    for k in range(d):
        model.add_constraint(w[k] <= beta[k])
        model.add_constraint(w[k] >= - beta[k])
    solution = model.solve()
    return np.array(solution.get_values(w))
```

$\Downarrow$



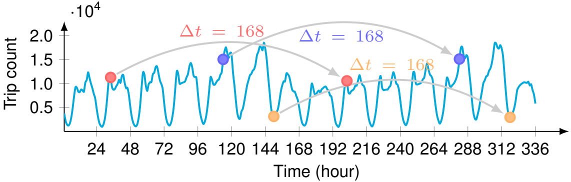
https://github.com/xinychen/integers

Solution Quality → Better Interpretability?

- Sparse autoregression

$$\min_{\boldsymbol{w}\geq 0} \ \|\tilde{\boldsymbol{x}}-\boldsymbol{A}\boldsymbol{w}\|_2^2 \quad \text{s.t. } \|\boldsymbol{w}\|_0 \leq \tau$$

- Subspace pursuit (SP) sometimes fails



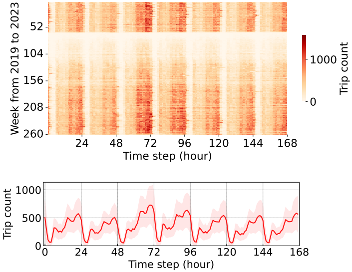
Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

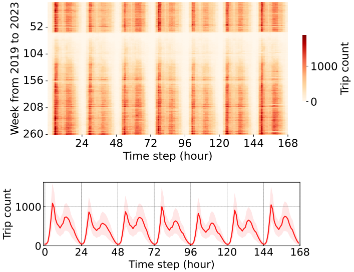
$$\underbrace{\boldsymbol{w} = (\cdots, \underbrace{0.02}_{k=53}, \cdots, \underbrace{0.96}_{k=168})^\top}_{\text{obj. } = 8.32\times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{\boldsymbol{w} = (\underbrace{0.22}_{k=1}, \cdots, \underbrace{0.77}_{k=168})^\top}_{\clubsuit \text{ obj. } = 6.25\times 10^7 \text{ (MIP)}}$$

John F. Kennedy International Airport

- Daily & weekly periodicity: **dropoff** $>$ **pickup** trips at JFK airport
  - Pickup trips are relevant to flight delay, baggage claim, and other factors.
  - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



Dropoff trips to airport

- Sparse coefficient vectors (sparsity $\tau = 3$):

$$\boldsymbol{w} = (\underbrace{0.31}_{k=1}, \cdots, \underbrace{0.28}_{k=24}, \cdots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \boldsymbol{w} = (\underbrace{0.18}_{k=1}, \cdots, \underbrace{0.35}_{k=24}, \cdots, \underbrace{0.47}_{k=168})^\top$$

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N:

$$\underbrace{\min_{\{\boldsymbol{w}_n\}_{n=1}^{N},\,\boldsymbol{\beta}}}_{(N+1)d\text{ decision var.}} \quad \underbrace{\sum_{n=1}^{N}\|\tilde{\boldsymbol{x}}_n-\boldsymbol{A}_n\boldsymbol{w}_n\|_2^2}_{\text{multivariate time series}}$$

$$\text{s.t.}\quad \underbrace{0\leq\boldsymbol{w}_n\leq\boldsymbol{\beta},}_{\text{bounds being either }0\text{ or }1}\qquad \underbrace{\|\boldsymbol{\beta}\|_1\leq\tau,}_{\text{sparsity of }\boldsymbol{\beta}}\quad \boldsymbol{\beta}\in\{0,1\}^d$$

- How to handle millions of time series (e.g., $N\geq 10^6$)?

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N:

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- How to handle millions of time series (e.g., $N\geq 10^6$)?

- Two-stage optimization (♣):

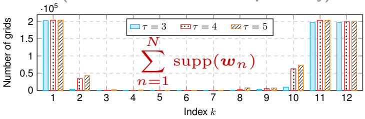
❶ **Learn sparsity patterns in $\boldsymbol{\beta}\in\{0,1\}^d$**

$$\min_{\boldsymbol{w},\,\boldsymbol{\beta}}\ \underbrace{\operatorname{tr}(\boldsymbol{w}\boldsymbol{w}^\top\boldsymbol{P})}_{\text{quadratic}}-\underbrace{2\boldsymbol{w}^\top\boldsymbol{q}}_{\text{linear}}$$

$$\text{s.t. } 0\leq\boldsymbol{w}\leq\boldsymbol{\beta},\quad \|\boldsymbol{\beta}\|_1\leq\tau$$

Global sparsity verification

(Tested N time series independently)



204k monthly NA temperature time series

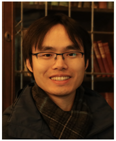
❷ **Quadratic programming** with index set $\Omega=\operatorname{supp}(\boldsymbol{\beta})$

$$\boldsymbol{w}_n := \arg\min_{\boldsymbol{w}\geq 0}\ \|\tilde{\boldsymbol{x}}_n-\boldsymbol{A}_n\boldsymbol{w}\|_2^2\quad \text{s.t. } w_k=0,\ \forall k\notin\Omega$$

Climate Seasonality Patterns

(arXiv:2506.22895)

- ❒ North America temperature
- ❒ Sea surface temperature
- ❒ Climate variable seasonality
- ❒ Spatiotemporal patterns



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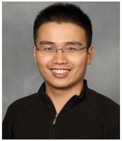

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Understanding Climate Systems

Quantify yearly seasonality by $\{w_{m,n,\gamma,k}\}$ at index $k = 12$

$$\min_{\{\boldsymbol{w}_{m,n,\gamma}\},\, \boldsymbol{\beta}} \; \overset{\text{latitude}}{\sum_{m=1}^{M}} \overset{\text{longitude}}{\sum_{n=1}^{N}} \overset{\text{decade}}{\sum_{\gamma=1}^{\delta}} \left\| \overset{\text{monthly temperature}}{\tilde{\boldsymbol{x}}_{m,n,\gamma} - \boldsymbol{A}_{m,n,\gamma}} \boldsymbol{w}_{m,n,\gamma} \right\|_2^2$$

$$\text{s.t. } 0 \leq \boldsymbol{w}_{m,n,\gamma} \leq \boldsymbol{\beta}$$

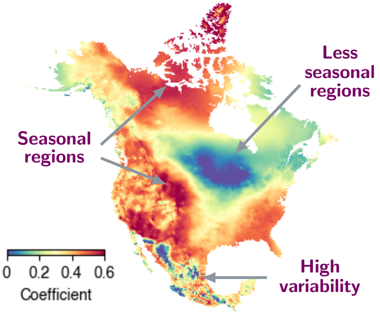
$$\|\boldsymbol{\beta}\|_1 \leq \tau$$

sparsity constraint

$$\boldsymbol{\beta} \in \{0,1\}^d$$

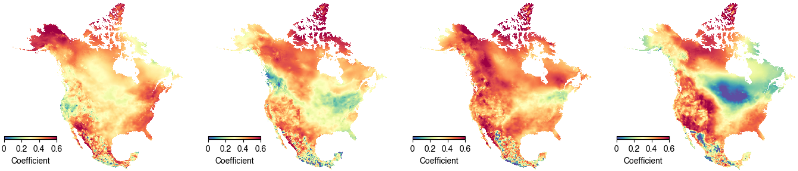
binary decision var.

North America Temperature

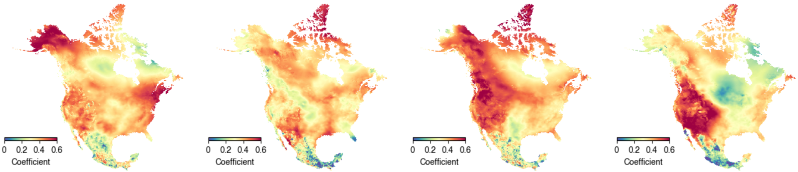


Yearly temperature **seasonality** pattern in 2010s

North America Temperature



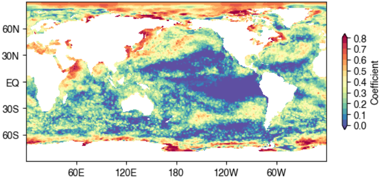
tmin, 1980s | tmin, 1990s | tmin, 2000s | tmin, 2010s



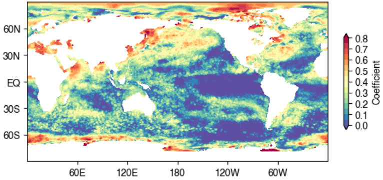
tmax, 1980s | tmax, 1990s | tmax, 2000s | tmax, 2010s

- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 3$)
  1. Stronger yearly seasonality in high-latitude areas
  2. Less seasonal temperature in south areas (e.g., Mexico)
  3. Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

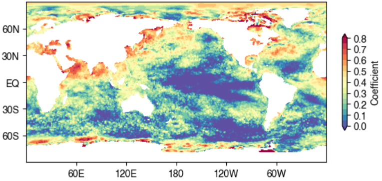
Sea Surface Temperature



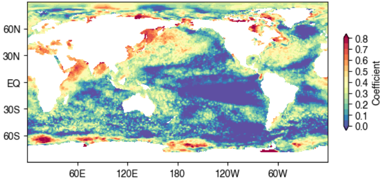
1980s



1990s



2000s



2010s

- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 3$)
  1. The areas of El Nino events are less seasonal/predictable
  2. Arctic becomes less seasonal/predictable in the past 20 years

Human Mobility Regularity

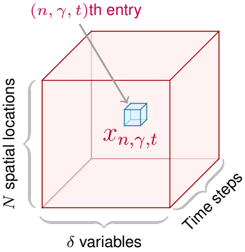
Applications and Case Studies

(arXiv:2508.03747)

- ❒ Daily & weekly periodicity
- ❒ NYC & Chicago ridesharing
- ❒ Multi-modal mobility
- ❒ Network resilience
- ❒ Post-pandemic recovery
- ❒ Metro passenger flow

Envisioning Human Mobility

- Human mobility data



spatial location
variable
hourly trip data

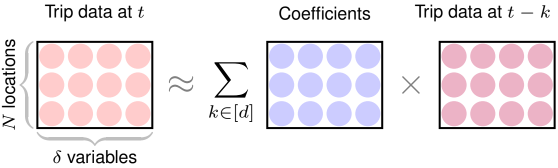
$$\min_{\{\boldsymbol{w}_{n,\gamma}\},\,\boldsymbol{\beta}} \sum_{n=1}^{N}\sum_{\gamma=1}^{\delta} \|\tilde{\boldsymbol{x}}_{n,\gamma} - \boldsymbol{A}_{n,\gamma}\boldsymbol{w}_{n,\gamma}\|_2^2$$

$$\text{s.t. } 0 \leq \boldsymbol{w}_{n,\gamma} \leq \boldsymbol{\beta}$$

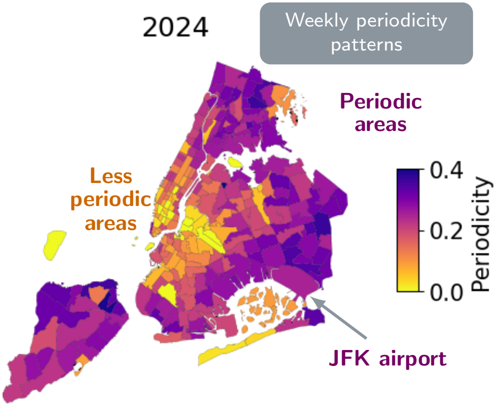
$$\|\boldsymbol{\beta}\|_1 \leq \tau \quad \text{sparse}$$

$$\boldsymbol{\beta} \in \{0,1\}^d \quad \text{binary}$$

- Quantify **weekly periodicity** by $\{w_{n,\gamma,k}\}$ at index $k = 168$



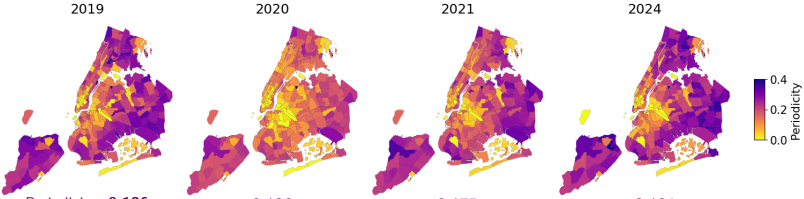
NYC Ridesharing



Hourly trip time series of 265 pickup areas

NYC Ridesharing

Weekly periodicity patterns



Periodicity: **0.186** | **0.136** | **0.175** | **0.194**

⇑ Pandemic disruption

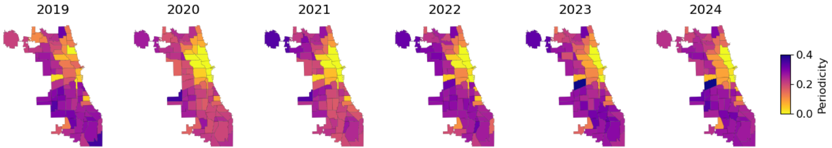
Weekly periodicity ↓

Less periodic areas ↑

⇑ Recovering

Chicago Ridesharing

Weekly periodicity patterns



Periodicity: 0.214 0.175 0.187 0.207 0.214 0.212

↑ Pandemic disruption

Weekly periodicity ↓

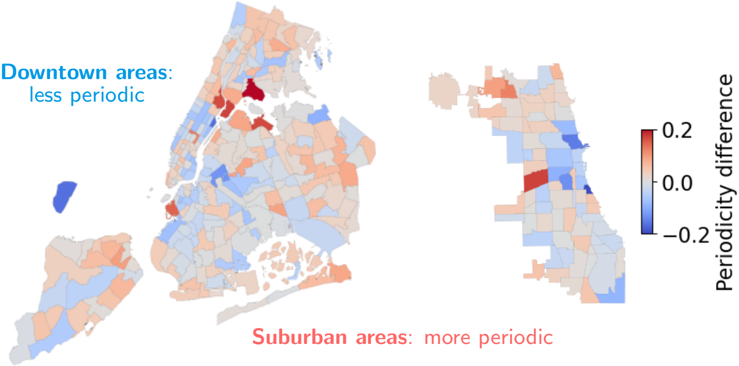
Less periodic areas ↑

↑ Recovering

Hourly trip time series of 77 pickup areas

Post-Pandemic Recovery

2024's periodicity **minus** 2019's periodicity



Downtown areas: less periodic

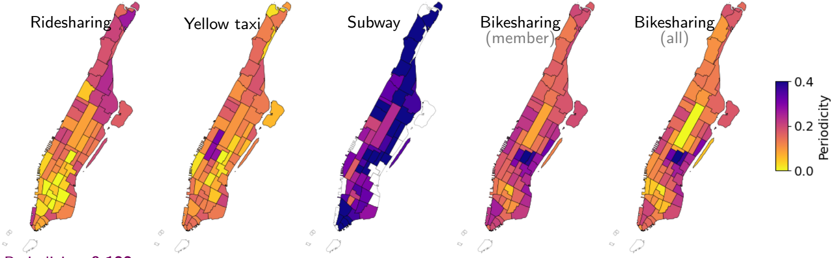
Suburban areas: more periodic

Weekly Periodicity of Manhattan Mobility

Mobility data of 2024

North areas:
Ridesharing > yellow taxi

Overall:
Subway > others



Periodicity: **0.122** **0.120** **0.363** **0.191** **0.151**

South areas:
Yellow taxi > Ridesharing

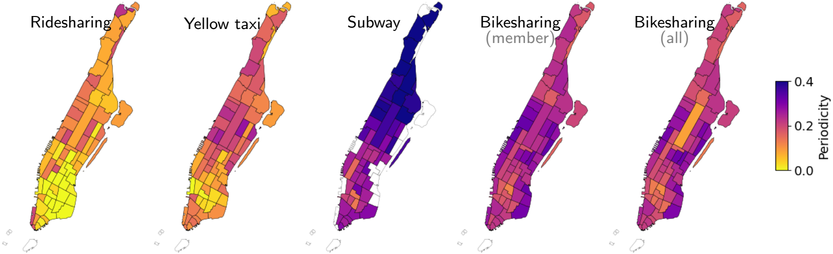
Remarkable weekly periodicity difference between membership and all travels

Daily Periodicity of Manhattan Mobility (Weekdays)

Mobility data of 2024

Overall:
Yellow taxi > ridesharing

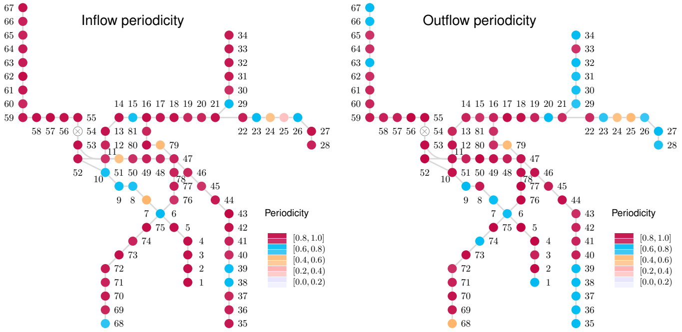
Subway:
North areas > south areas



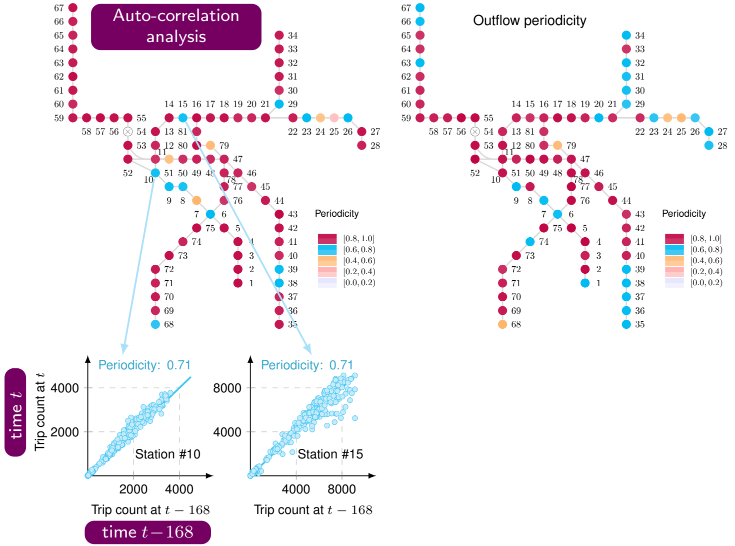
Periodicity: **0.072** **0.127** **0.305** **0.232** **0.214**

Marginal daily periodicity difference between membership and all travels

Hangzhou Metro Passenger Flow

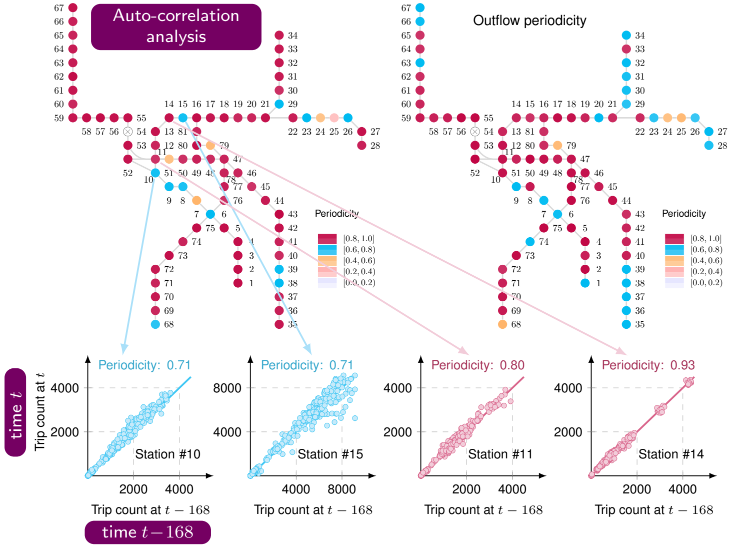


Hangzhou Metro Passenger Flow



“Closeness” to the anti-diagonal curve $y = x$

Hangzhou Metro Passenger Flow



"Closeness" to the anti-diagonal curve $y = x$

Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/ints.pdf

About me:

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