



Quantifying Time Series Periodicity with Interpretable Machine Learning

Climate Systems & Urban Human Mobility

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Atlanta, USA

Spatiotemporal Time Series Data

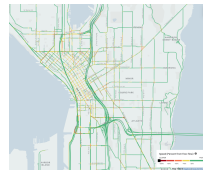
- Transport & mobility & climate application scenarios



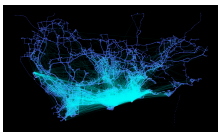
Highway (Portland)



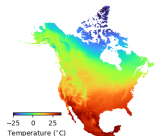
Uber movement (NYC)



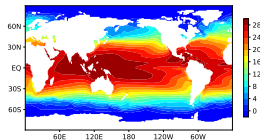
Uber movement (Seattle)



Taxi trajectory (Shenzhen)



Temperature (NA)

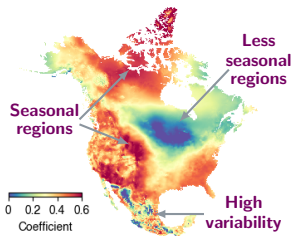


Temperature (sea surface)

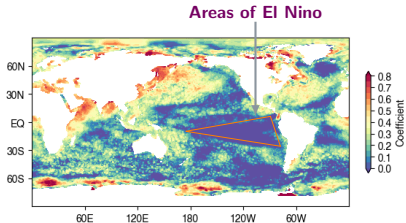
- Challenges: Sparsity, high-dimensionality, multi-dimensionality, heavy tails, irregular sampling, and time-varying systems

Motivation

Yearly temperature seasonality patterns in 2010s



North America



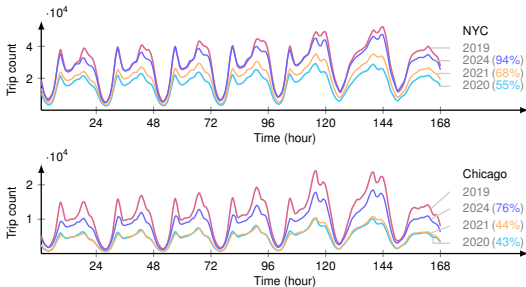
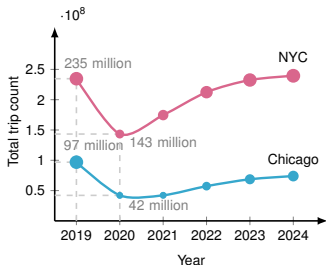
Global sea surface

What motivate us most about periodicity?

- ❶ **Monitoring climate systems:** Empirically measure the periodicity of climate variables (e.g., temperature, precipitation).
- ❷ **Discovering spatiotemporal patterns:** Identify periodicity pattern shift and special climate events.

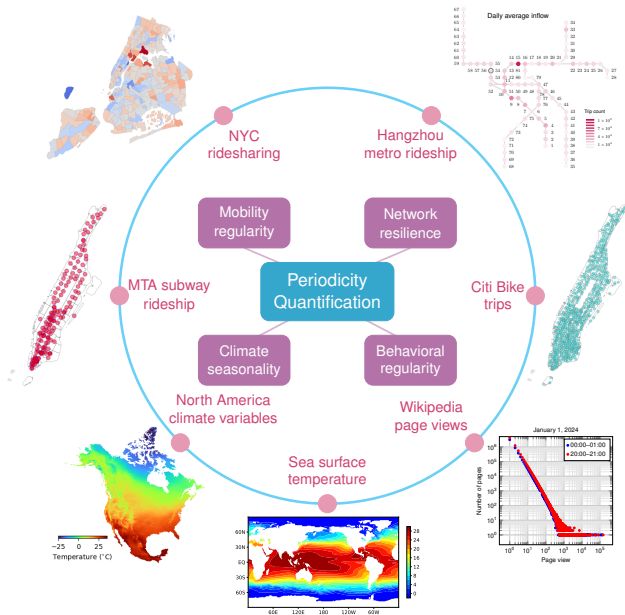
Motivation

Ridesharing trip data



What motivate us most about periodicity?

- ➊ **Resilience and stability of systems:** Empirically measure the periodicity and predictability of urban systems.
- ➋ **Optimization of transport systems:** Optimize resources (e.g., public transit, taxi, ridesharing, micromobility) to meet transport demand efficiently.
- ➌ **Design of sustainable transport & infrastructure:** Implement energy-efficient solutions (e.g., congestion pricing) tailored to peak hours.



Interpretable Time Series Autoregression



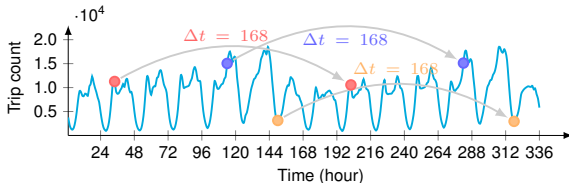
<https://github.com/xinychen/integers>

- ☐ Interpretable ML
- ☐ ℓ_0 -norm optimization
- ☐ Climate system seasonality
- ☐ Sparse autoregression
- ☐ Mixed-integer programming
- ☐ Human mobility regularity

Valorizing Autoregression

- Time series autoregression on $\mathbf{x} \in \mathbb{R}^T$ with order $d \in \mathbb{Z}^+$

$$\mathbf{w} := \arg \min_{\mathbf{w}} \sum_{t=d+1}^T \left(x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2$$



Periodicity of hourly rideshare trip time series

- Sparse** coefficient vector $\mapsto$ **Interpretability**?

$$\underbrace{\mathbf{w}}_{\text{sparsity } \|\mathbf{w}\|_0 \triangleq 3} = (\underbrace{0.33}_{k=1}, 0, \dots, 0, \underbrace{0.20}_{k=167}, \underbrace{0.46}_{k=168})^\top \in \mathbb{R}^{168}$$

Sparse Autoregression

- Identify the dominant auto-correlations

◦ $\tau \in \mathbb{Z}^+$: Upper bound of the number of nonzero entries in $\mathbf{w} \in \mathbb{R}^d$

$$\begin{array}{c} \tilde{\mathbf{x}} \\ \begin{array}{|c|} \hline x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ \hline \end{array} \end{array} \approx \begin{array}{c} \mathbf{A} \\ \begin{array}{|c|c|c|c|} \hline x_4 & x_3 & x_2 & x_1 \\ x_5 & x_4 & x_3 & x_2 \\ x_6 & x_5 & x_4 & x_3 \\ x_7 & x_6 & x_5 & x_4 \\ x_8 & x_7 & x_6 & x_5 \\ \hline \end{array} \end{array} \times \begin{array}{c} \mathbf{w} \\ \begin{array}{|c|} \hline w_1 \\ w_2 \\ w_3 \\ w_4 \\ \hline \end{array} \end{array} \left. \vphantom{\begin{array}{c} \mathbf{w} \\ \begin{array}{|c|} \hline w_1 \\ w_2 \\ w_3 \\ w_4 \\ \hline \end{array} \right\} \|\mathbf{w}\|_0 \leq \tau} \right\} \|\mathbf{w}\|_0 \leq \tau$$

$$\begin{aligned} \mathbf{w} &:= \arg \min_{\|\mathbf{w}\|_0 \leq \tau} \sum_{t=d+1}^T \left(x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2 \\ &= \arg \min_{\|\mathbf{w}\|_0 \leq \tau} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \end{aligned}$$

- ℓ_0 -norm optimization is NP-hard
- Formulate it as a mixed-integer programming

◦ Introduce binary decision variables $\beta \in \{0, 1\}^d$

$$\begin{array}{ll} \min_{\mathbf{w}} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 & \min_{\mathbf{w}, \beta} \|\tilde{\mathbf{x}} - \mathbf{A}\mathbf{w}\|_2^2 \\ \text{s.t. } \underbrace{\|\mathbf{w}\|_0 \leq \tau}_{\clubsuit \text{ sparsity of } \mathbf{w}} & \iff \text{s.t. } \underbrace{-\beta \leq \mathbf{w} \leq \beta}_{\text{bounds being either 0 or } \pm 1} \quad \underbrace{\|\beta\|_1 \leq \tau}_{\clubsuit \text{ sparsity of } \beta} \end{array}$$

ℓ_0 -Norm Optimization

Sparse Autoregression Done Right

$$\min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\sum_{t=d+1}^T \left(x_t - \sum_{k=1}^d w_k x_{t-k} \right)^2}_{\text{Time series autoregression}} \quad \text{s.t.} \quad \underbrace{-\beta_k \leq w_k \leq \beta_k}_{\text{Lower and upper bounds}}, \quad \underbrace{\sum_{k=1}^d \beta_k \leq \tau}_{\clubsuit \text{ Sparsity}}, \quad \underbrace{\beta_k \in \{0, 1\}}_{\text{Binary variable}}$$

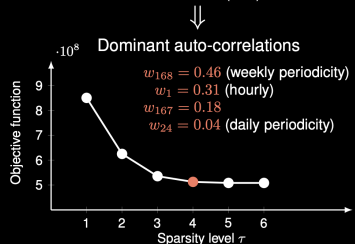
- $\mathbf{w} \in \mathbb{R}^d$: Auto-correlations
- $\boldsymbol{\beta} \in \{0, 1\}^d$: Sparsity pattern
- $d = 168$: Autoregression order

```

1 import numpy as np
2 from docplex.mp.model import Model
3
4 def sparse_ar(x, d, tau):
5     model = Model('Sparse Autoregression')
6     T = x.shape[0]
7     w = [model.continuous_var(name = f'w_{k}') for k in range(d)]
8     beta = [model.binary_var(name = f'beta_{k}') for k in range(d)]
9     model.minimize(model.sum([x[t] - model.sum(w[k] * x[t - k - 1]
10                                     for k in range(d))) ** 2
11                             for t in range(d, T)])
12     model.add_constraint(model.sum(beta[k] for k in range(d)) <= tau)
13     for k in range(d):
14         model.add_constraint(w[k] <= beta[k])
15         model.add_constraint(w[k] >= -beta[k])
16     solution = model.solve()
17     return np.array(solution.get_values(w))

```

<https://github.com/xinychen/integers>

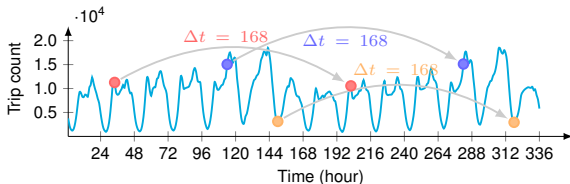


Solution Quality $\rightarrow$ Better Interpretability?

- Sparse autoregression

$$\min_{w \geq 0} \|\tilde{x} - Aw\|_2^2 \quad \text{s.t.} \quad \|w\|_0 \leq \tau$$

- Subspace pursuit (SP) sometimes fails



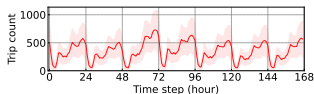
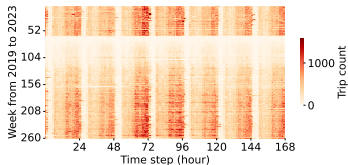
Periodicity of ridesharing trip time series

- Exact solution w/ mixed-integer programming (MIP)
- An intuitive example (sparsity $\tau = 2$):

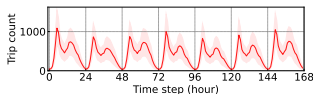
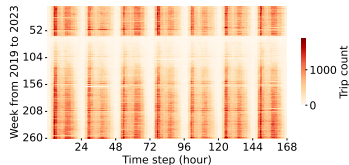
$$\underbrace{w = (\dots, \underbrace{0.02}_{k=53}, \dots, \underbrace{0.96}_{k=168})^\top}_{\text{obj.} = 8.32 \times 10^7 \text{ (SP)}} \quad \text{vs.} \quad \underbrace{w = (\underbrace{0.22}_{k=1}, \dots, \underbrace{0.77}_{k=168})^\top}_{\clubsuit \text{ obj.} = 6.25 \times 10^7 \text{ (MIP)}}$$

John F. Kennedy International Airport

- **Daily & weekly periodicity: dropoff > pickup** trips at JFK airport
 - Pickup trips are relevant to flight delay, baggage claim, and other factors.
 - Dropoff trips to airport are highly related to flight schedules.



Pickup trips from airport



Dropoff trips to airport

- Sparse coefficient vectors (sparsity $\tau = 3$):

$$\mathbf{w} = (\underbrace{0.31}_{k=1}, \dots, \underbrace{0.28}_{k=24}, \dots, \underbrace{0.41}_{k=168})^\top \quad \text{vs.} \quad \mathbf{w} = (\underbrace{0.18}_{k=1}, \dots, \underbrace{0.35}_{k=24}, \dots, \underbrace{0.47}_{k=168})^\top$$

High-Dimensional Sparse Autoregression

- On high-dimensional time series with a large N :

$$\begin{aligned}
 & \min_{\underbrace{\{\mathbf{w}_n\}_{n=1}^N, \boldsymbol{\beta}}_{(N+1)d \text{ decision var.}}} \underbrace{\sum_{n=1}^N \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}_n\|_2^2}_{\text{multivariate time series}} \\
 & \text{s.t.} \quad \underbrace{0 \leq \mathbf{w}_n \leq \boldsymbol{\beta}}_{\text{bounds being either 0 or 1}}, \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sparsity of } \boldsymbol{\beta}}, \quad \boldsymbol{\beta} \in \{0, 1\}^d
 \end{aligned}$$

- How to handle **millions of** time series (e.g., $N \geq 10^6$)?

High-Dimensional Sparse Autoregression

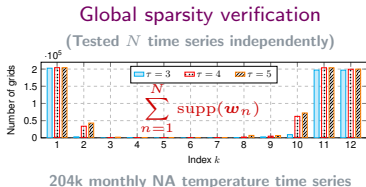
- On high-dimensional time series with a large N :

$$\begin{aligned}
 & \min_{\underbrace{\{\mathbf{w}_n\}_{n=1}^N, \boldsymbol{\beta}}_{(N+1)d \text{ decision var.}}} \underbrace{\sum_{n=1}^N \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}_n\|_2^2}_{\text{multivariate time series}} \\
 & \text{s.t.} \quad \underbrace{0 \leq \mathbf{w}_n \leq \boldsymbol{\beta}}_{\text{bounds being either 0 or 1}}, \quad \underbrace{\|\boldsymbol{\beta}\|_1 \leq \tau}_{\text{sparsity of } \boldsymbol{\beta}}, \quad \boldsymbol{\beta} \in \{0, 1\}^d
 \end{aligned}$$

- How to handle **millions of** time series (e.g., $N \geq 10^6$)?
- Two-stage optimization (♣):

- Learn sparsity patterns in $\boldsymbol{\beta} \in \{0, 1\}^d$

$$\begin{aligned}
 & \min_{\mathbf{w}, \boldsymbol{\beta}} \underbrace{\text{tr}(\mathbf{w} \mathbf{w}^\top \mathbf{P})}_{\text{quadratic}} - \underbrace{2 \mathbf{w}^\top \mathbf{q}}_{\text{linear}} \\
 & \text{s.t. } 0 \leq \mathbf{w} \leq \boldsymbol{\beta}, \quad \|\boldsymbol{\beta}\|_1 \leq \tau
 \end{aligned}$$



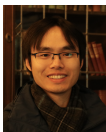
- Quadratic programming with index set $\Omega = \text{supp}(\boldsymbol{\beta})$

$$\mathbf{w}_n := \arg \min_{\mathbf{w} \geq 0} \|\tilde{\mathbf{x}}_n - \mathbf{A}_n \mathbf{w}\|_2^2 \quad \text{s.t. } w_k = 0, \forall k \notin \Omega$$

Climate Seasonality Patterns

(arXiv:2506.22895)

- ❑ North America temperature
- ❑ Climate variable seasonality
- ❑ Sea surface temperature
- ❑ Spatiotemporal patterns



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Jinhua Zhao
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Understanding Climate Systems

Quantify yearly seasonality by $\{w_{m,n,\gamma,k}\}$ at index $k = 12$

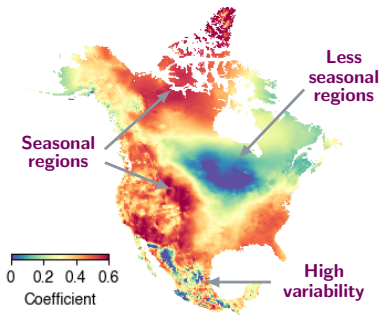
$$\begin{aligned} \min_{\{w_{m,n,\gamma}\}, \beta} \quad & \sum_{m=1}^M \sum_{n=1}^N \sum_{\gamma=1}^{\delta} \|\tilde{x}_{m,n,\gamma} - A_{m,n,\gamma} w_{m,n,\gamma}\|_2^2 \\ \text{s.t.} \quad & 0 \leq w_{m,n,\gamma} \leq \beta \\ & \|\beta\|_1 \leq \tau \\ & \beta \in \{0, 1\}^d \end{aligned}$$

longitude
latitude
decade
monthly temperature

sparsity constraint

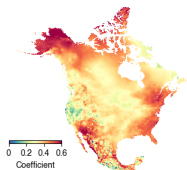
binary decision var.

North America Temperature

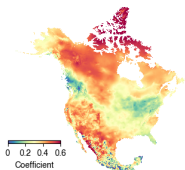


Yearly temperature seasonality pattern in 2010s

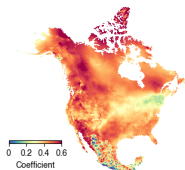
North America Temperature



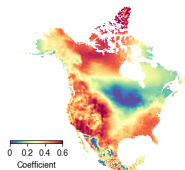
tmin, 1980s



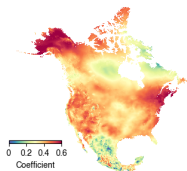
tmin, 1990s



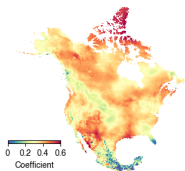
tmin, 2000s



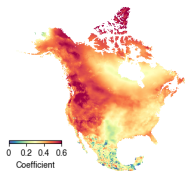
tmin, 2010s



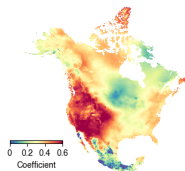
tmax, 1980s



tmax, 1990s



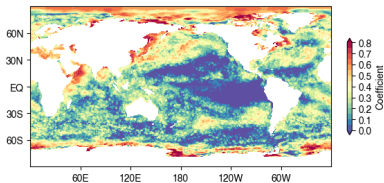
tmax, 2000s



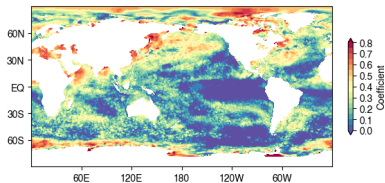
tmax, 2010s

- Identify yearly periodicity at $k = 12$ from temperature data ($\tau = 3$)
 - ① Stronger yearly seasonality in high-latitude areas
 - ② Less seasonal temperature in south areas (e.g., Mexico)
 - ③ Seasonality patterns in 2000s & 2010s are different from 1980s & 1990s

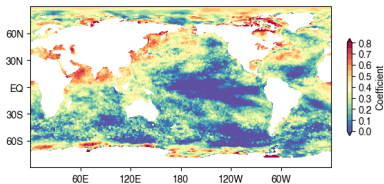
Sea Surface Temperature



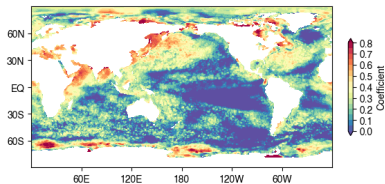
1980s



1990s



2000s



2010s

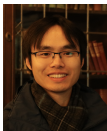
- Identify yearly periodicity at $k = 12$ from SST data ($\tau = 3$)
 - ① The areas of **El Nino** events are less seasonal/predictable
 - ② **Arctic** becomes less seasonal/predictable in the past 20 years

Human Mobility Regularity

Applications and Case Studies

(arXiv:2508.03747)

- ❑ Daily & weekly periodicity
- ❑ NYC & Chicago ridesharing
- ❑ Multi-modal mobility
- ❑ Network resilience
- ❑ Post-pandemic recovery
- ❑ Metro passenger flow



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Qi Wang
NEU



Yunhan Zheng
MIT → PKU



Nina Cao
MIT



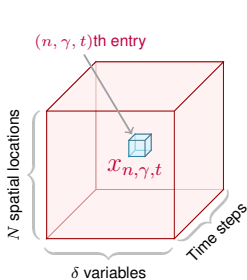
HanQin Cai
UCF



Jinhua Zhao
MIT

Envisioning Human Mobility

- Human mobility data



$$\begin{aligned}
 & \min_{\{w_{n, \gamma}\}, \beta} \sum_{n=1}^N \sum_{\gamma=1}^{\delta} \|\tilde{x}_{n, \gamma} - A_{n, \gamma} w_{n, \gamma}\|_2^2 \\
 & \text{s.t. } 0 \leq w_{n, \gamma} \leq \beta \\
 & \quad \|\beta\|_1 \leq \tau \quad \leftarrow \text{sparse} \\
 & \quad \beta \in \{0, 1\}^d \quad \leftarrow \text{binary}
 \end{aligned}$$

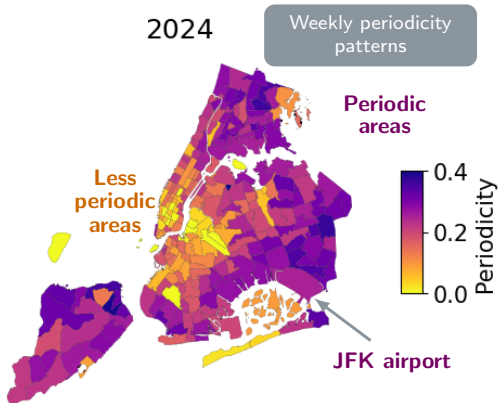
spatial location variable

hourly trip data

- Quantify **weekly periodicity** by $\{w_{n, \gamma, k}\}$ at index $k = 168$

$$\begin{array}{ccc}
 \text{Trip data at } t & & \text{Coefficients} \quad \text{Trip data at } t - k \\
 \begin{array}{c} N \text{ locations} \\ \left\{ \begin{array}{|c|} \hline \text{3x3 grid of red circles} \\ \hline \end{array} \right. \\ \delta \text{ variables} \end{array} & \approx \sum_{k \in [d]} & \begin{array}{c} \left\{ \begin{array}{|c|} \hline \text{3x3 grid of blue circles} \\ \hline \end{array} \right\} \times \begin{array}{|c|} \hline \text{3x3 grid of pink circles} \\ \hline \end{array} \end{array}
 \end{array}$$

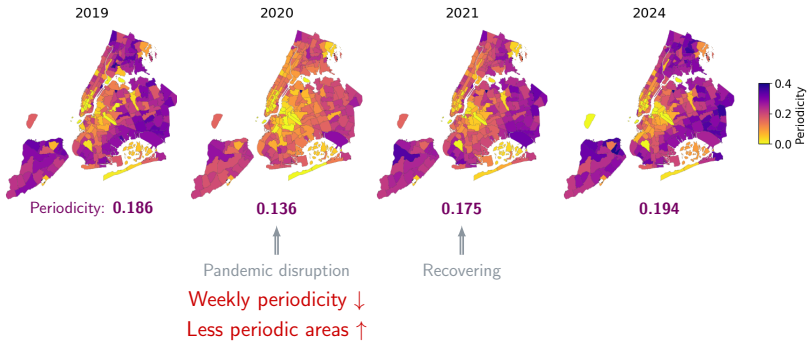
NYC Ridesharing



Hourly trip time series of 265 pickup areas

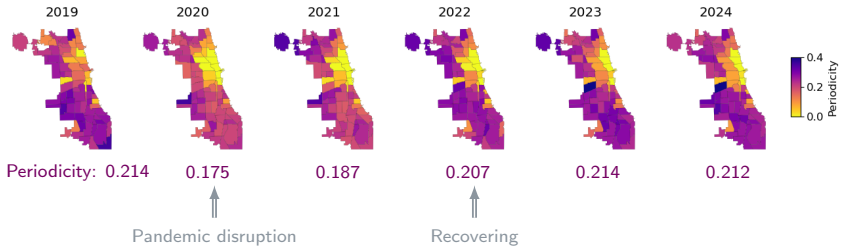
NYC Ridesharing

Weekly periodicity patterns



Chicago Ridesharing

Weekly periodicity patterns



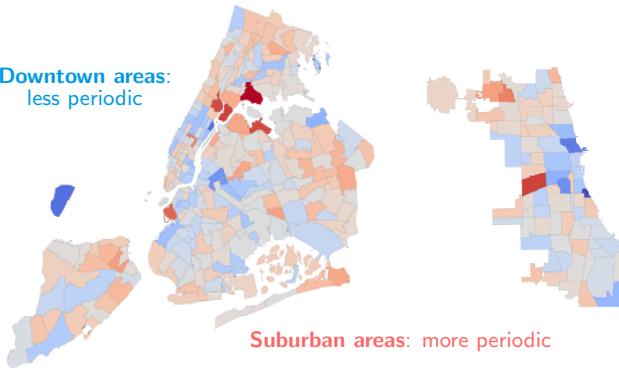
Weekly periodicity ↓
Less periodic areas ↑

Hourly trip time series of 77 pickup areas

Post-Pandemic Recovery

2024's periodicity **minus** 2019's periodicity

Downtown areas:
less periodic



Suburban areas: more periodic

Weekly Periodicity of Manhattan Mobility

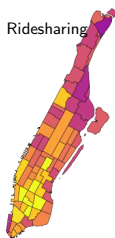
Mobility data of 2024

North areas:

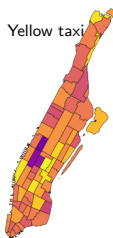
Ridesharing > yellow taxi

Overall:

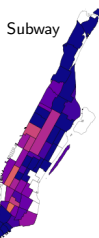
Subway > others



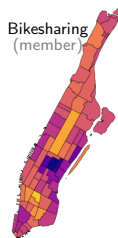
Periodicity: **0.122**



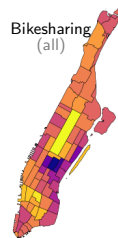
0.120



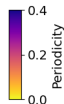
0.363



0.191



0.151



South areas:

Yellow taxi > Ridesharing

Remarkable weekly periodicity difference
between membership and all travels

Daily Periodicity of Manhattan Mobility (Weekdays)

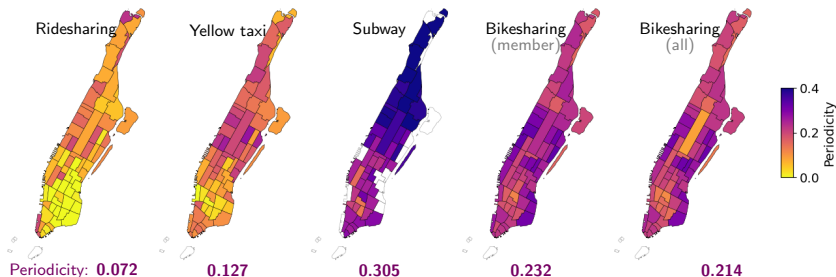
Mobility data of 2024

Overall:

Yellow taxi > ridesharing

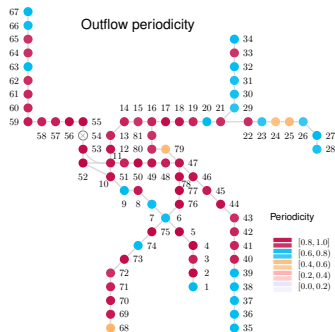
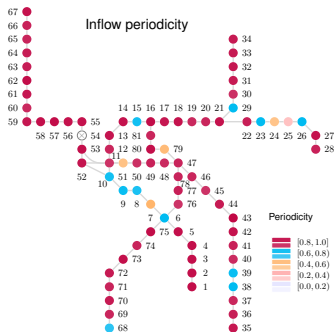
Subway:

North areas > south areas

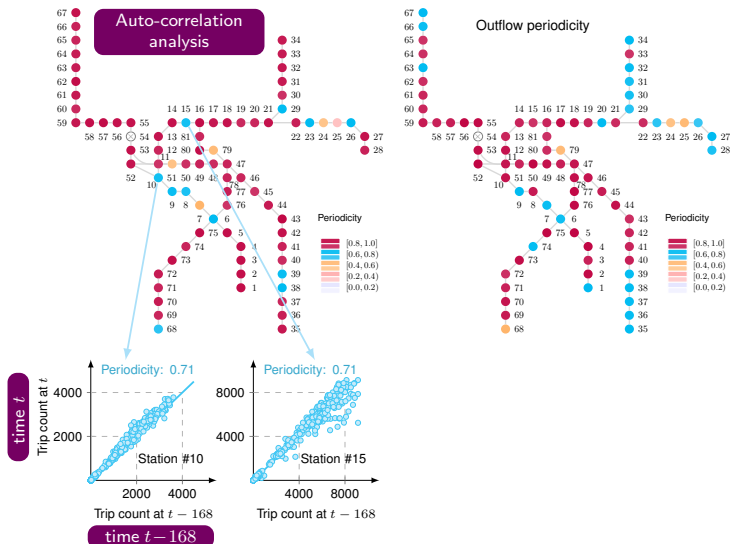


Marginal daily periodicity difference
between membership and all travels

Hangzhou Metro Passenger Flow

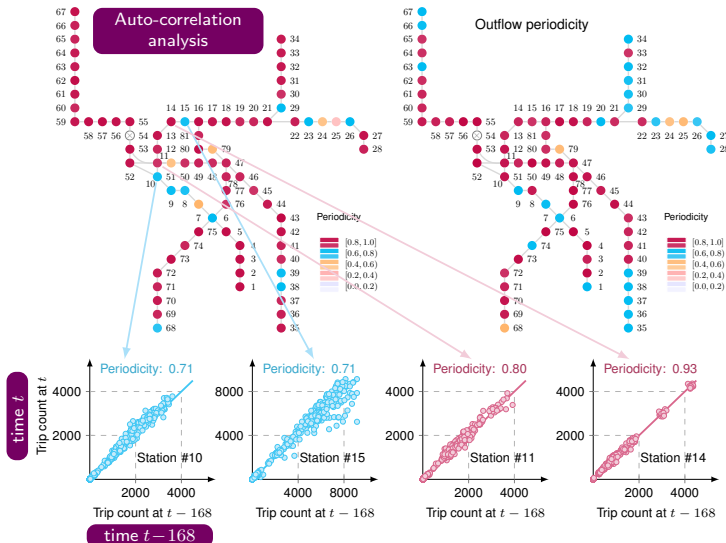


Hangzhou Metro Passenger Flow



"Closeness" to the anti-diagonal curve $y = x$

Hangzhou Metro Passenger Flow



“Closeness” to the anti-diagonal curve $y = x$



Thanks for your attention!

Any Questions?

Slides: <https://xinychen.github.io/slides/informs25.pdf>

About me:

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