Laplacian Convolutional Representation for Traffic Time Series Imputation

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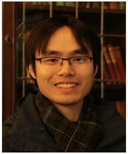
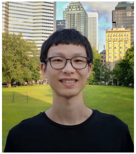


Xinyu Chen
PolyMtl → MIT



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Open source:

- Spatiotemporal data modeling initiative: https://spatiotemporal-data.github.io

Outline

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  Time Series Imputation
  Speed Field Reconstruction
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  Revisit Circular Convolution
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- **Global Trend Modeling**
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  Solution Algorithm
  Empirical Time Complexity
- **Experiments**
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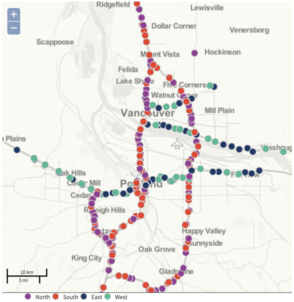
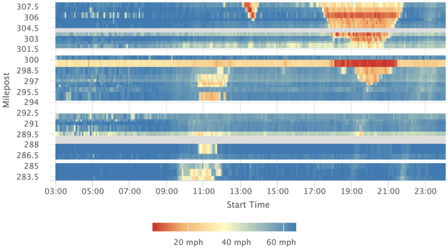
  Speed Field Reconstruction
- **Conclusion**

Motivation

- Portland highway traffic flow data[1]



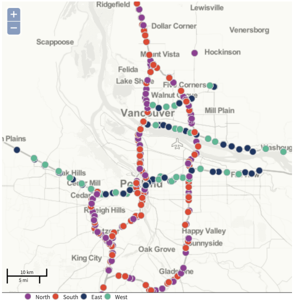
Highway network & N sensors



Traffic speed field

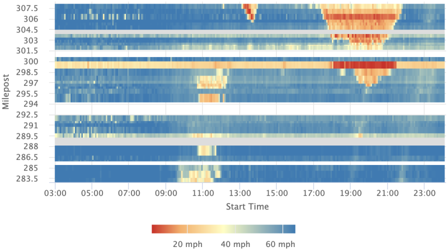
Motivation

- Portland highway traffic flow data[1]



Highway network & N sensors

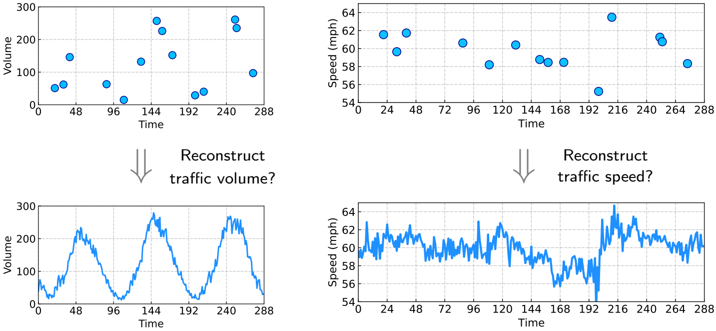
$\boldsymbol{X} \in \mathbb{R}^{N \times T}$



Traffic speed field

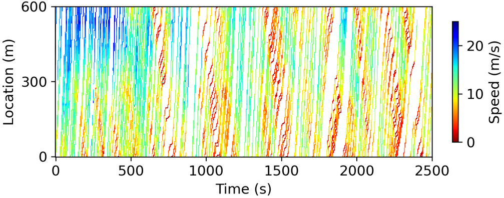
[1]https://portal.its.pdx.edu/home

Motivation

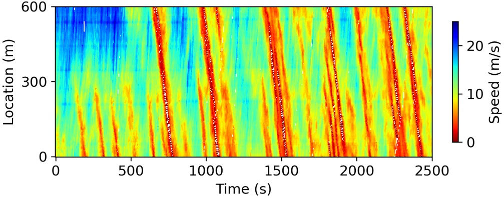


- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

Motivation



200-by-500 matrix (NGSIM) $\Downarrow$ Reconstruct speed field from 20% sparse trajectories?

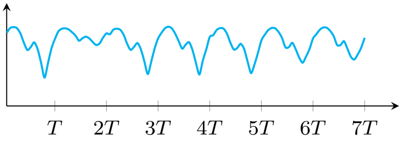


- How to learn from sparse spatiotemporal data?
- How to characterize spatial/temporal local dependencies?

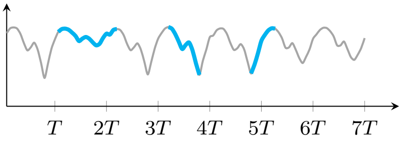
Motivation

Sparse time series imputation

- Global trends (e.g., long-term quasi-seasonality & daily/weekly rhythm)



- Local trends (e.g., short-term time series trends)

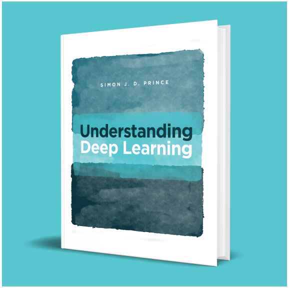


Modeling global & local trends simultaneously?

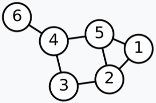
Revisit Laplacian Matrix

- Graph Laplacian matrix:

$$\boldsymbol{L} = \underbrace{\boldsymbol{D}}_{\text{degree matrix}} - \underbrace{\boldsymbol{A}}_{\text{adjacency matrix}}$$

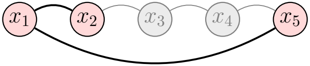


https://udlbook.github.io/udlbook/

Labelled graph	Degree matrix	Adjacency matrix	Laplacian matrix
	$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$

— "*Laplacian matrix*" on Wikipedia

- Intuition of Laplacian matrix.



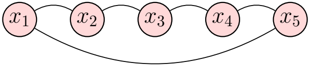
Undirected and circulant graph

$$\xrightarrow{\text{Modeling}}$$

$$\boldsymbol{L} = \begin{bmatrix} \mathbf{2} & -1 & 0 & 0 & -1 \\ \mathbf{-1} & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ \mathbf{-1} & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Intuition of Laplacian matrix.



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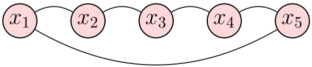
(Circulant) Laplacian matrix

Revisit Circular Convolution

Reformulate Laplacian regularization with circular convolution.

- Intuition of (circulant) Laplacian matrix.



Undirected and circulant graph

$$\xrightarrow{\text{Modeling}}$$

$$\boldsymbol{L}=\begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

- Laplacian kernel: $\boldsymbol{\ell}=(2,-1,0,0,-1)^\top$.

$$\boldsymbol{L}\boldsymbol{x}=\begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}\star\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}=\boldsymbol{\ell}\star\boldsymbol{x}$$

where $\star$ denotes the ciruclar convolution.

- Local trend modeling via (Laplacian) temporal regularization:

$$\mathcal{R}(\boldsymbol{x})=\frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2=\frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2$$

"... The circulant graph has an adjacency matrix that is a circulant matrix."

— "*Circulant graph*" on Wikipedia

Reformulate Laplacian Regularization

- Define Laplacian kernel:

$$\boldsymbol{\ell} \triangleq (\underbrace{2\tau}_{\text{degree}}, \underbrace{-1, \cdots, -1}_{\tau}, 0, \cdots, 0, \underbrace{-1, \cdots, -1}_{\tau})^\top \in \mathbb{R}^T$$

for any time series $\boldsymbol{x} = (x_1, \ldots, x_T)^\top \in \mathbb{R}^T$.

[2] It refers to the Convolution theorem.

Reformulate Laplacian Regularization

- Define Laplacian kernel:

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- Local trend modeling via (Laplacian) temporal regularization:

$$\mathcal{R}_\tau(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{L}\boldsymbol{x}\|_2^2 = \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$$

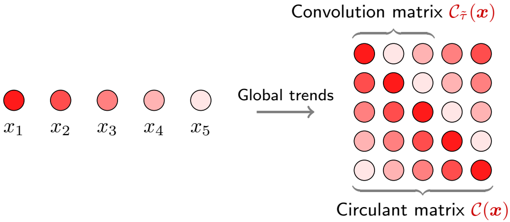
- Property with discrete Fourier transform (denoted by $\mathcal{F}(\cdot)$)[2]:

$$\mathcal{R}_\tau(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 = \underbrace{\frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell}) \circ \mathcal{F}(\boldsymbol{x})\|_2^2}_{\text{w/ FFT in } \mathcal{O}(T\log T) \text{ time}}$$

[2] It refers to the Convolution theorem.

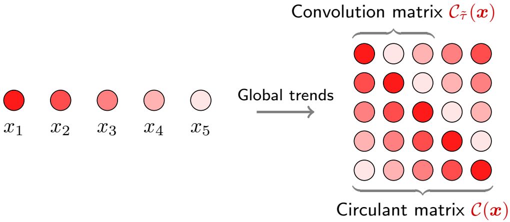
Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



Global Trend Modeling

Circulant matrix $\mathcal{C}(\boldsymbol{x})$ vs. convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})$



- Circulant/Convolution nuclear norm minimization (w/ $\|\mathcal{C}(\boldsymbol{x})\|_* = \|\mathcal{F}(\boldsymbol{x})\|_1$)
  - A balance between global and local trends modeling?

CircNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* \quad \text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

ConvNNM (Liu'22, Liu & Zhang'23)

Estimating $\boldsymbol{x}$:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_* \quad \text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

on data $\boldsymbol{y}$ w/ observed index set Ω.

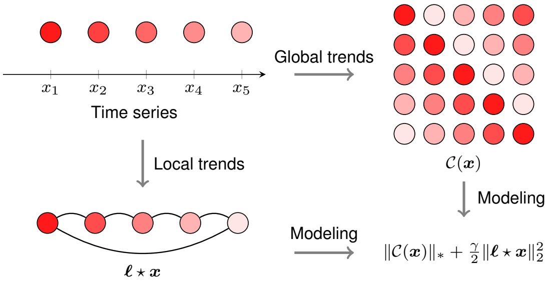
Global + Local Trends?

Laplacian Convolutional Representation (LCR)

For any partially observed time series $\boldsymbol{y} \in \mathbb{R}^T$ with observed index set Ω, LCR utilizes circulant matrix and Laplacian kernel to characterize global and local trends in time series, respectively, i.e.,

$$\min_{\boldsymbol{x}} \; \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$



Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned}\min_{\boldsymbol{x}}\ & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon\end{aligned}$$

$$\Longrightarrow \quad \begin{aligned}\min_{\boldsymbol{x}}\ & \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2}_{\text{global + local modeling}} + \underbrace{\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z}-\boldsymbol{y})\|_2^2}_{\text{constraint to regularization}} \\ \text{s.t. } & \boldsymbol{z}=\boldsymbol{x}\end{aligned}$$

Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned}\min_{\boldsymbol{x}}\ & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 \\ \text{s.t. } & \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon\end{aligned}$$

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"The alternating direction method of multipliers (ADMM) is an algorithm that solves convex optimization problems by breaking them into smaller pieces, each of which are then easier to handle."

— Source: https://stanford.edu/~boyd/admm.html

Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned}\min_{\boldsymbol{x}}\ & \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2^2 \\ \text{s.t. } & \boldsymbol{z}=\boldsymbol{x}\end{aligned}$$

- Augmented Lagrangian function[3]:

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) = \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}\|_2^2 + \langle\boldsymbol{w},\boldsymbol{x}-\boldsymbol{z}\rangle + \frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z}-\boldsymbol{y})\|_2^2$$

[3] w/ Lagrange multiplier $\boldsymbol{w}\in\mathbb{R}^T$ and inner product $\langle\boldsymbol{x},\boldsymbol{y}\rangle=\boldsymbol{x}^\top\boldsymbol{y}$.

Laplacian Convolutional Representation

- LCR model:

$$\begin{aligned}\min_{\boldsymbol{x}}&\ \|\mathcal{C}(\boldsymbol{x})\|_*+\frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2+\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2^2\\ \text{s.t. }&\ \boldsymbol{z}=\boldsymbol{x}\end{aligned}$$

- Augmented Lagrangian function[3]:

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w})=\|\mathcal{C}(\boldsymbol{x})\|_*+\frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2+\frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}\|_2^2+\langle\boldsymbol{w},\boldsymbol{x}-\boldsymbol{z}\rangle+\frac{\eta}{2}\|\mathcal{P}_\Omega(\boldsymbol{z}-\boldsymbol{y})\|_2^2$$

- The ADMM scheme:

$$\begin{cases}\boldsymbol{x}:=\arg\min_{\boldsymbol{x}}\ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Nuclear norm minimization)}\\ \boldsymbol{z}:=\arg\min_{\boldsymbol{z}}\ \mathcal{L}(\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}) & \text{(Closed-form solution)}\\ \quad=\frac{1}{\lambda+\eta}\mathcal{P}_\Omega(\lambda\boldsymbol{x}+\boldsymbol{w}+\eta\boldsymbol{y})+\frac{1}{\lambda}\mathcal{P}_\Omega^\perp(\lambda\boldsymbol{x}+\boldsymbol{w}) & \\ \boldsymbol{w}:=\boldsymbol{w}+\lambda(\boldsymbol{x}-\boldsymbol{z}) & \text{(Standard update)}\end{cases}$$

- Optimize $\boldsymbol{x}$?

$$\underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*=\|\mathcal{F}(\boldsymbol{x})\|_1}_{\text{property of circulant matrix}}\quad\&\quad\underbrace{\frac{1}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2=\frac{1}{2T}\|\mathcal{F}(\boldsymbol{\ell})\circ\mathcal{F}(\boldsymbol{x})\|_2^2}_{\text{property of circular convolution}}$$

[3] w/ Lagrange multiplier $\boldsymbol{w}\in\mathbb{R}^T$ and inner product $\langle\boldsymbol{x},\boldsymbol{y}\rangle=\boldsymbol{x}^\top\boldsymbol{y}$.

Laplacian Convolutional Representation

- Optimize $\boldsymbol{x}$ via FFT (in $\mathcal{O}(T\log T)$ time):

$$\begin{aligned} \boldsymbol{x} &:= \arg\min_{\boldsymbol{x}} \; \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}+\boldsymbol{w}/\lambda\|_2^2 \\ \Longrightarrow \hat{\boldsymbol{x}} &:= \arg\min_{\hat{\boldsymbol{x}}} \; \|\hat{\boldsymbol{x}}\|_1 + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}}\circ\hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{z}}+\hat{\boldsymbol{w}}/\lambda\|_2^2 \end{aligned}$$

where we introduce $\{\hat{\boldsymbol{\ell}},\hat{\boldsymbol{x}},\hat{\boldsymbol{z}},\hat{\boldsymbol{w}}\}\triangleq\mathcal{F}\{\boldsymbol{\ell},\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}\}$ (i.e., FFT).

Laplacian Convolutional Representation

- Optimize $\boldsymbol{x}$ via FFT (in $\mathcal{O}(T\log T)$ time):

$$\begin{aligned} \boldsymbol{x} &:= \arg\min_{\boldsymbol{x}}\ \|\mathcal{C}(\boldsymbol{x})\|_{*} + \frac{\gamma}{2}\|\boldsymbol{\ell}\star\boldsymbol{x}\|_2^2 + \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{z}+\boldsymbol{w}/\lambda\|_2^2 \\ \Longrightarrow \hat{\boldsymbol{x}} &:= \arg\min_{\hat{\boldsymbol{x}}}\ \|\hat{\boldsymbol{x}}\|_1 + \frac{\gamma}{2T}\|\hat{\boldsymbol{\ell}}\circ\hat{\boldsymbol{x}}\|_2^2 + \frac{\lambda}{2T}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{z}}+\hat{\boldsymbol{w}}/\lambda\|_2^2 \end{aligned}$$

where we introduce $\{\hat{\boldsymbol{\ell}},\hat{\boldsymbol{x}},\hat{\boldsymbol{z}},\hat{\boldsymbol{w}}\}\triangleq\mathcal{F}\{\boldsymbol{\ell},\boldsymbol{x},\boldsymbol{z},\boldsymbol{w}\}$ (i.e., FFT).

ℓ_1-norm Minimization in Complex Space (Liu & Zhang'23)

For any optimization problem in the form of ℓ_1-norm minimization in complex space:

$$\min_{\hat{\boldsymbol{x}}}\ \|\hat{\boldsymbol{x}}\|_1 + \frac{\delta}{2}\|\hat{\boldsymbol{x}}-\hat{\boldsymbol{h}}\|_2^2$$

with complex-valued $\hat{\boldsymbol{x}},\hat{\boldsymbol{h}}\in\mathbb{C}^T$ and weight parameter δ, element-wise, the solution is given by

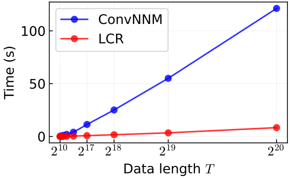
$$\hat{x}_t := \frac{\hat{h}_t}{|\hat{h}_t|}\cdot\max\{0,|\hat{h}_t|-1/\delta\}, t=1,\ldots,T.$$

Laplacian Convolutional Representation

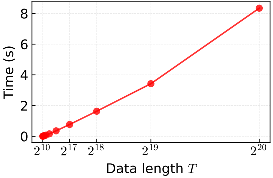
Empirical time complexity

On the synthetic data $\boldsymbol{y} \in \mathbb{R}^{T}$ with $T \in \{2^{10}, 2^{11}, \ldots, 2^{20}\}$

- Ours: **LCR**
  - An FFT implementation in $\mathcal{O}(T \log T)$
  - The logarithmic factor $\log T$ makes the FFT highly efficient
- Baseline: **ConvNNM** (Liu'22, Liu & Zhang'23)
  - Convolution matrix $\mathcal{C}_{\tilde{\tau}}(\boldsymbol{y}) \in \mathbb{R}^{T \times \tilde{\tau}}$ with kernel size $\tilde{\tau} = 2^4$
  - Singular value thresholding in $\mathcal{O}(\tilde{\tau}^2 T)$

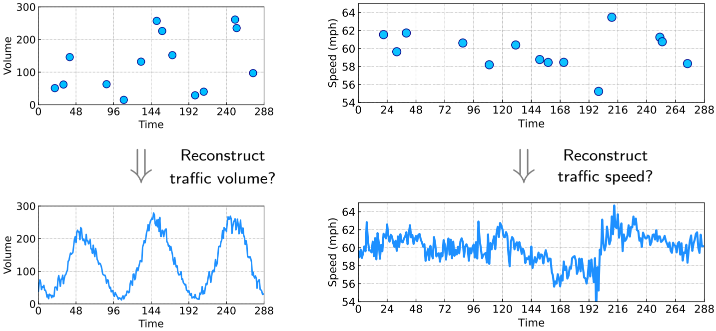


ConvNNM vs. LCR



LCR

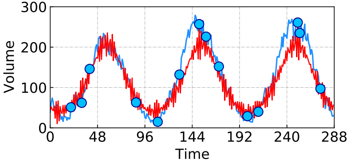
Experiments



- How to utilize the global trends of traffic time series?
- How to produce local consistency of traffic data?

Experiments

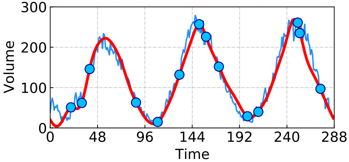
- Substantial performance gains?



CircNNM:

$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* \quad \text{s.t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

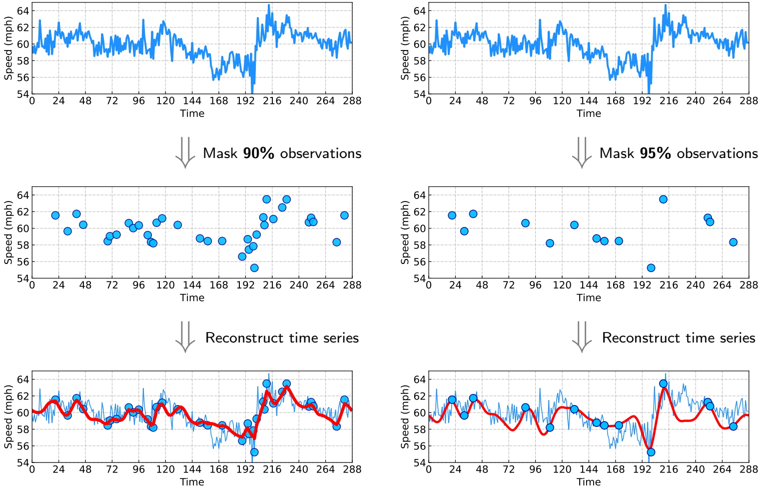
$\Downarrow$ Plus **local** time series trends



LCR:

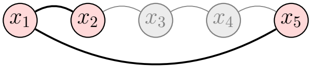
$$\min_{\boldsymbol{x}} \ \|\mathcal{C}(\boldsymbol{x})\|_* + \frac{\gamma}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2 \quad \text{s.t.} \ \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon$$

Experiments



Experiments

- The start data points and end data points are connected?



Undirected and circulant graph

$$\xrightarrow{\text{Modeling}} \quad \boldsymbol{L} = \begin{bmatrix} \mathbf{2} & -1 & 0 & 0 & -1 \\ \mathbf{-1} & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ \mathbf{-1} & 0 & 0 & -1 & 2 \end{bmatrix}$$

(Circulant) Laplacian matrix

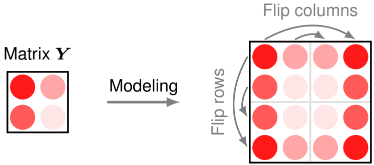
- Flipping operation on $\boldsymbol{x} \in \mathbb{R}^5$:

$$\boldsymbol{x}_{\text{new}} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{J}\boldsymbol{x} \end{bmatrix} = (\underbrace{x_1, x_2, x_3, x_4, x_5,}_{\text{original time series}} \ \underbrace{x_5, x_4, x_3, x_2, x_1}_{\text{flipped time series}})^\top \in \mathbb{R}^{10}$$

where $\boldsymbol{J} \in \mathbb{R}^{5\times 5}$ is the exchange matrix.

Speed field reconstruction[4]

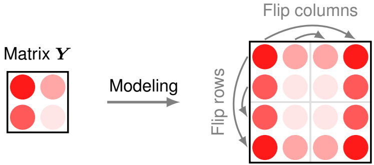
- Flipping operation on a matrix:



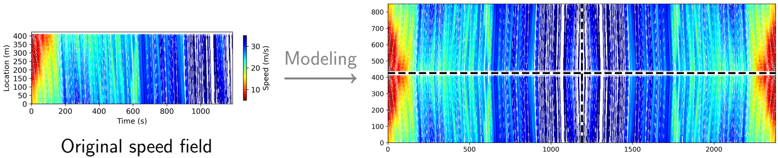
[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Speed field reconstruction[4]

- Flipping operation on a matrix:



- Flipping operation on a speed field of vehicular traffic flow:



Original speed field

Flipped speed field

[4] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

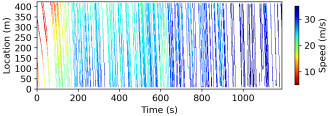
Experiments

Speed field reconstruction[5]

- Scenario: Mask trajectories of 70% vehicles
- LCR-2D on partially observed $\boldsymbol{Y} \in \mathbb{R}^{N \times T}$:

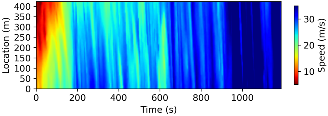
$$\min_{\boldsymbol{X}} \; \underbrace{\|\mathcal{C}(\boldsymbol{X})\|_*}_{\text{global trend}} + \frac{\gamma}{2} \underbrace{\|(\boldsymbol{\ell}_s \boldsymbol{\ell}^\top) \star \boldsymbol{X}\|_F^2}_{\text{local trend}}$$

$$\text{s.t. } \|\mathcal{P}_\Omega(\boldsymbol{X} - \boldsymbol{Y})\|_F \leq \epsilon$$

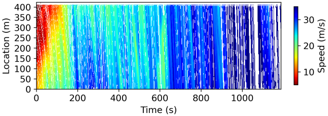


Sparse speed field

LCR-2D



Reconstructed speed field



Ground-truth speed field

[5] Highway Drone (HighD) dataset at https://www.highd-dataset.com/

Contributions

Matrix nuclear norm ($\|\boldsymbol{X}\|_*$) minimization

Candès & Recht'09

Singular value thresholding

Cai et al.'10

Truncated nuclear norm ($\|\boldsymbol{X}\|_{r,*}, r \in \mathbb{Z}^+$) minimization

Zhang et al.'12

Hu et al.'12

Tensor nuclear norm ($\|\boldsymbol{\mathcal{X}}\|_*$) minimization

Liu et al.'13

Generalized nonconvex nonsmooth low-rank minimization

Lu et al.'14

Tensor nuclear norm minimization with linear transform

Lu et al.'19

Low-rank Hankel matrix/tensor ($\mathcal{H}_\tau(\cdot)$) completion

Yokota et al.'18

Sedighin et al.'20

Cai et al.'21

Yamamoto et al.'22

Circulant/Convolution nuclear norm ($\|\mathcal{C}(\boldsymbol{x})\|_*$ or $\|\mathcal{C}_{\tilde{\tau}}(\boldsymbol{x})\|_*$) minimization

Liu'22

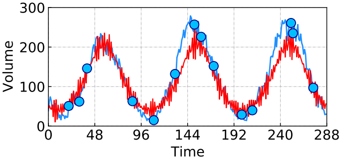
Liu & Zhang'23

(Ours) **LCR**:

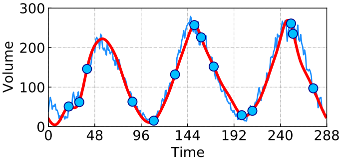
✓ Local trend modeling

✓ An FFT implementation

Vision & Insight



⇓ Plus **local** time series trends



Highlights:

- Rethinking the importance of local trend modeling in traffic data imputation tasks.
- Finding a unified global and local trend modeling framework whose optimization can be efficiently solved by FFT:

$$\min_{\boldsymbol{x}} \ \underbrace{\|\mathcal{C}(\boldsymbol{x})\|_*}_{\text{global}} + \frac{\gamma}{2} \underbrace{\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2}_{\text{local}}$$

$$\text{s.t.} \ \ \|\mathcal{P}_\Omega(\boldsymbol{x} - \boldsymbol{y})\|_2 \leq \epsilon$$

Concluding Remarks

- **(Starting point)** How to impute traffic time series?
  - ✓ Low-rank models ✓ Temporal regularization
- **(Solution)** Time series trend modeling in the low-rank framework?
  - Global time series trend modeling (low-rank model):

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \|\mathcal{C}(\boldsymbol{x})\|_* \\ \text{s. t.} \quad & \|\mathcal{P}_\Omega(\boldsymbol{x}-\boldsymbol{y})\|_2 \leq \epsilon \end{aligned}$$

  - Local time series trend modeling (temporal regularization):

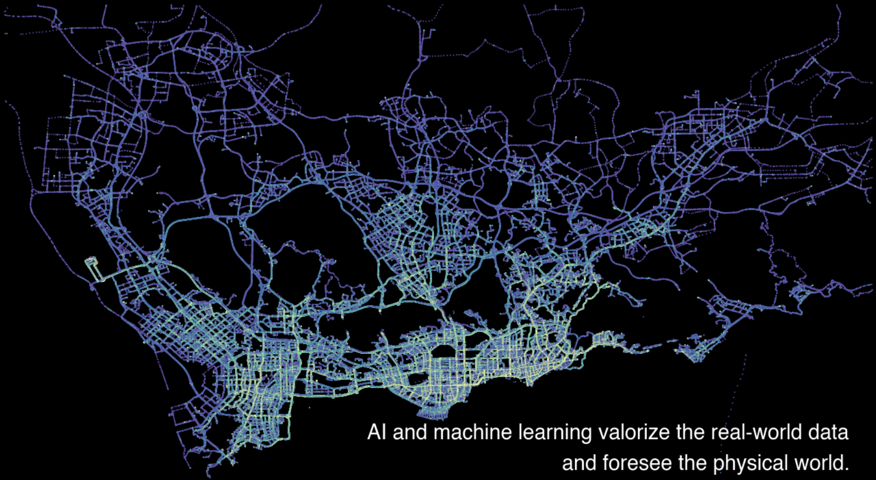
$$\mathcal{R}_\tau(\boldsymbol{x}) = \frac{1}{2}\|\boldsymbol{\ell} \star \boldsymbol{x}\|_2^2$$

- **(Highlight)** A unified framework with the FFT implementation.

References

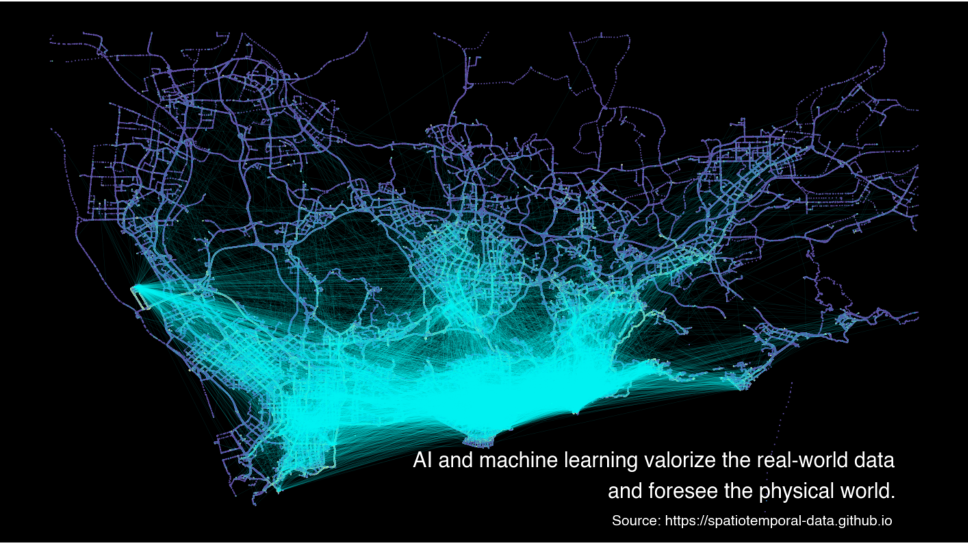
A short list:

- **[Liu'22]** G. Liu (2022). Time series forecasting via learning convolutionally low-rank models. IEEE Transactions on Information Theory, 68(5): 3362–3380.
- **[Liu & Zhang'23]** G. Liu and W. Zhang (2023). Recovery of future data via convolution nuclear norm minimization. IEEE Transactions on Information Theory, 69(1): 650–665.



AI and machine learning valorize the real-world data
and foresee the physical world.

Source: https://spatiotemporal-data.github.io


AI and machine learning valorize the real-world data
and foresee the physical world.
Source: https://spatiotemporal-data.github.io

Thanks for your attention!

Any Questions?

Slides: https://xinychen.github.io/slides/LCR24.pdf

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