CGN 3405: Applied Numerical Methods for Civil Engineering

Topic: Modeling and Errors

Note: Please use Colab to write your Python code. When you are ready to submit, please check out the TA's Announcement (see Webcourses) in terms of downloading "`.ipynb`." TA will run your Python codes and check the results.

Submission Format: `.ipynb` Only

Q1: Polynomial Function Approximation (15 points)

Use first- through third-order Taylor series expansions to predict $f(3)$ for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point at $x = 1$. Compute the true percent relative error $\varepsilon = |\text{Approximation} - \text{True Value}|$ for each approximation.

Requirements:

- Use `import numpy as np`.
- Write clear, commented code.
- Print the results.

Q2: Approximating $\ln(1+x)$ (20 points)

The Taylor series for the natural logarithm $\ln(1+x)$ centered at $a=0$ is:

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

Programming Task:

1. Calculate the Absolute Error for $\ln(1.2)$ as you increase the order n from 1 to 10.
2. Calculate the Absolute Error for $\ln(1.5)$ as you increase the order n from 1 to 10.

Requirements:

- Use `import numpy as np`.
- Write clear, commented code.
- Print the results.

Q3: Arctangent Maclaurin Series (20 points)

The Maclaurin series (a Taylor series expansion about 0) for the arctangent of x is defined for $|x| \leq 1$ as:

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$$

Programming Task:

- Calculate the true value of $f(x) = \tan^{-1}(x)$ for $x = \pi/6$.
- Use the orders $n = 0, 1, 2, 3$ to estimate $\tan^{-1}(\pi/6)$.
- For each approximation above, calculate the absolute error $\varepsilon = |\text{Approximation} - \text{True Value}|$.

Requirement:

- Use `import numpy as np`.
- Write clean, commented code.
- Print the results.